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Hydromechanic Coefficients for Calculating Time Domain Motions of Cutter Suction Dredges by Cummins Equations

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Abstract

Software package DREDMO of the Delft Hydraulics predicts the behaviour of seagoing cutter dredges in near-shore conditions, which behaviour can be important with respect to the construction of the dredge and the assessment of downtime. The motion behaviour of the seagoing dredge has been described by non-linear Cummins Equations, which have to be solved in the time domain.
The required input data on hydromechanic coefficients, retardation functions and frequency domain wave load series can be derived with a newly developed pre-processing program SEWAY-D. This program, based on the frequency domain ship motions program SEAWAY, creates the hydromechanic input data file for DREDMO with a minimum risk on human input errors and makes DREDMO much more accessible for less-specialist users.
This report describes the underlying hydromechanic theory of the new SEAWAY-D and DREDMO releases. Also, comparisons with frequency domain results have been given.

1 Introduction

The prediction of the behaviour of cutter dredges in near-shore conditions can be important with respect to the construction of the dredge and the assessment of downtime. To be able to make downtime predictions, the Delft Hydraulics together with the Laboratory of Soil Movement and the Ship Hydromechanics Laboratory (both are laboratories of the Delft University of Technology) developed the computer code DREDMO [13] in the early 80's. The behaviour of the seagoing dredge has been described by non-linear so-called Cummins Equations, which have to be solved in the time domain. These equations require hydromechanic

coefficients, retardation functions and wave load series as input, together with geometric and operational data of the ship and the operational working method.
The required input data – hydromechanic coefficients, retardation functions and frequency domain wave loads - have to be derived from model experiments or from calculated data by a suitable ship motion computer program. However, the creation of this hydromechanic input data file appeared to be very much time consuming. Besides this, DREDMO and its pre- and post-processing programs were running on main frames, and specialists were required to operate the software.

In 1984, Delft Hydraulics decided to develop a PC version of DREDMO, to promote the use of their commercial package.

In the late 80's the author had developed computer code SEAWAY [6,8], a frequency domain ship motions program based on the strip theory, for calculating the wave-induced motions and resulting mechanic loads of mono-hull ships moving forward with six degrees of freedom in a seaway.
The potential coefficients and wave loads are calculated for infinite and finite water depths. Added resistance, shearing forces and bending and torsion moments can be calculated too. Linear(ised) springs and free surface anti-rolling tanks, bilge keels and other anti-rolling devices can be included. Several wave spectra definitions can been used.

Computed data have been validated with results of other computer programs and experimental data. Based on these validation studies - and experiences, obtained during an intensive use of the program by the author, students and over 30 institutes and industrial users - it is expected that this program is free of significant errors.

In 1990, the author used relevant parts of SEAWAY to create a pre-processing program for DREDMO. The Laboratory of Soil Movement had defined the formats of the hydromechanic input data of DREDMO. This pre-processing program is called SEAWAY-D and makes DREDMO more accessible for less-specialist users.

In 1991, the Laboratory of Soil Movement had completed their work on DREDMO with the delivery of PC pre- and post-processing programs and a user-interface, which minimises the risk on human input errors; see [13].

In 1992, an input control program SEAWAY-H, to check the input data of the offsets of the under water geometry of the ship, have been added to the DREDMO software package.

Since 1993, DREDMO calculations can be carried out too for ships with local symmetric twin-hull sections - as for instance appear at cutter suction dredges - provided that interaction effects between individual sections are not accounted for. Special attention has been paid to longitudinal jumps in the cross sections, fully submerged cross sections and (non)linear viscous roll damping coefficients. Improved definitions of the hydrodynamic potential masses at an infinite frequency and the wave loads have been added. Finally, many numerical routines have been improved.

All strip-theory algorithms are described in [9]. A user manual of SEAWAY-D, with an example of the input and the output files, has been given in [11].
This report contains a survey of the hydromechanic part of the underlying theory of DREDMO and SEAWAY-D and a validation of the calculated results.
A separately developed "stripped" version of DREDMO, called SEAWAY-T, has been used here to verify all algorithms in the time domain.

2 Equations of Motion

The co-ordinate systems are defined in Figure 1.

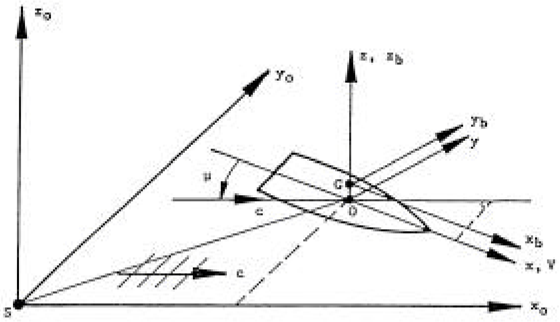

Figure 1 Co-ordinate System and Definitions

Three right-handed co-ordinate systems have been defined:

1. $G(x_b, y_b, z_b)$ connected to the ship, with G at the ship's centre of gravity, x_b in the ship's centre line in the forward direction, y_b in the ship's port side direction and z_b in the upward direction

2. $S(x_0, y_0, z_0)$ fixed in space, with S in the still water surface, x_0 in the direction of the wave propagation and z_0 in the upward direction.

3. $O(x, y, z)$ or $O(x_1, x_2, x_3)$ moving with the ship's speed, with O above or below the average position of the ship's centre of gravity, x or x_1 parallel to still water surface, y or x_2 parallel to still water surface and z or x_3 in the upward direction. The angular motions of the body about the body axes are denoted by: f, q and y or x_4, x_5 and x_6.

3 Frequency Domain Calculations

Based on Newton's second law of dynamics, the equations of motion of a floating object in a seaway are given by:

$$\sum_{j=1}^{6} M_{i,j} \cdot \ddot{x}_j = F_i$$

for i = 1, 2, … 6

in which:

$M_{i,j}$	6x6 matrix of solid mass and inertia of the body
$\ddot{x}_j$	acceleration of the body in direction j
F_i	sum of forces or moments acting in direction i

When defining a linear system with simple harmonic wave exciting forces and moments, defined by:

$$F_{w_i}(w,t) = F_{w_{ai}}(w) \cdot \cos\left(wt + e_{w_i}(w)\right)$$

The resulting simple harmonic displacements, velocities and accelerations are:

$$x_j(w,t) = x_{aj}(w) \cdot \cos(wt)$$
$$\dot{x}_j(w,t) = -w \cdot x_{aj}(w) \cdot \sin(wt)$$
$$\ddot{x}_j(w,t) = -w^2 \cdot x_{aj}(w) \cdot \cos(wt)$$

The hydromechanic forces and moments F_i, acting on the free floating object in waves, consist of:

- linear hydrodynamic reaction forces and moments expressed in terms with the hydrodynamic mass and damping coefficients:
$$-a_{i,j}(w) \cdot \ddot{x}_j(w,t) - b_{i,j}(w) \cdot \dot{x}_j(w,t)$$

- linear hydrostatic restoring forces and moments expressed in a term with a spring coefficient:
$$-c_{i,j} \cdot x_j(w,t)$$

With this, the linear equations of motion become:

$$\sum_{j=1}^{6} \left\{ M_{i,j} \cdot \ddot{x}_j \right\} =$$
$$\sum_{j=1}^{6} \left\{ -a_{i,j}(w) \cdot \ddot{x}_j(w,t) - b_{i,j}(w) \cdot \dot{x}_j(w,t) \right.$$
$$\left. -c_{i,j} \cdot x_j(w,t) \right\} + F_{w_{ai}}(w) \cdot \cos\left(wt + e_{w_i}(w)\right)$$
$$\text{for } i = 1, 2, \ldots 6$$

So:

$$\sum_{j=1}^{6} \left\{ \begin{array}{l} \left(M_{i,j} + a_{i,j}(w)\right) \cdot \ddot{x}_j(w,t) \\ + b_{i,j}(w) \cdot \dot{x}_j(w,t) + c_{i,j} \cdot x_j(w,t) \end{array} \right\} =$$

$$= F_{w_{ai}}(w) \cdot \cos\left(wt + e_{w_i}(w)\right)$$
$$\text{for } i = 1, 2, \ldots 6$$

The hydromechanic coefficients $a_{i,j}(w)$, $b_{i,j}(w)$ and $c_{i,j}$ and the wave load components $F_{w_{ai}}(w)$ can be calculated with existing 2-D or 3-D techniques.
For this, strip theory programs - like for instance the program SEAWAY [3] - can be used. According to the strip theory, the total hydromechanic coefficients and wave loads of the ship can be found easily by integrating the cross-sectional values over the ship length.

The strip theory is a slender body theory, so one should expect less accurate predictions for ships with low length to breadth ratios. However, experiments have shown that the strip theory appears to be remarkably effective for predicting the motions of ships with length to breadth ratios down to about 3.0 or sometimes even lower.
The strip theory is based upon the potential flow theory. This holds that viscous effects are neglected, which can deliver serious problems when predicting roll motions at resonance frequencies. In practice, viscous roll damping effects can be accounted for by experimental results or by empirical formulas.
The strip theory is based upon linearity. This means that the ship motions are supposed to be small, relative to the cross-sectional dimensions of the ship. Only hydrodynamic effects of the hull below the still water level are accounted for. So when parts of the ship go out of or into the water or when green water is shipped, inaccuracies can be expected. Also, the strip theory does not distinguish between alternative above water hull forms.
Nevertheless these limitations for zero forward speed, generally the strip theory appears to be a successful and practical theory for the calculation of the wave induced motions of a ship.

For the determination of the two-dimensional potential coefficients for sway, heave and roll motions of not fully submerged ship-like cross sections, the so-called 2-Parameter Lewis Transformation can map cross-sections conformally to the unit circle. Also the N-Parameter Close-Fit Conformal Mapping Method can be used for this.

The advantage of conformal mapping is that the velocity potential of the fluid around an arbitrary shape of a cross section in a complex plane can be derived from the more convenient circular cross section in another complex plane. In this manner hydrodynamic problems can be solved directly with the coefficients of the mapping function, as reported by Tasai [15,16].
The advantage of making use of the 2-Parameter Lewis Conformal Mapping Method is that the frequency-depending potential coefficients are a function of the breadth, the draught and the area of the cross section, only.

Another method is the Frank Method [2], also suitable for fully submerged cross sections. This method determines the velocity potential of a floating or a

submerged oscillating cylinder of infinite length by the integral equation method utilising the Green's function, which represents a pulsating source below the free surface.
To avoid so-called "irregular frequencies" in the operational frequency range of not fully submerged cross sections, each Frank section will be closed automatically at the free surface with a few extra points. This results into a shift of these irregular frequencies towards a higher frequency region.

The two-dimensional pitch and yaw coefficients follow from the heave and sway moments, respectively.

Finally, a method based on work published by Kaplan and Jacobs [12] and a longitudinal strip method has been used for the determination of the two-dimensional potential coefficients for the surge motion.

At the following sections, the hydromechanic coefficients and the wave loads for zero forward speed are given as they can be derived from the two-dimensional values, defined in a co-ordinate system with the origin O in the waterline.

The symbols, used here, are:

$M_{i,j}$	solid mass and inertia coefficients of the body
$m_{i,j}{}'(w)$	sectional hydrodynamic mass coefficient
$n_{i,j}{}'(w)$	sectional hydrodynamic damping coefficient
$F_{w_i}{}'(w)$	sectional wave exciting force or moment
$FK_{w_i}{}'(w)$	sectional Froude-Krylov force or moment
$\ddot{z}_{w_i}{}^{*}(w)$	equivalent orbital acceleration
$\dot{z}_{w_i}{}^{*}(w)$	equivalent orbital velocity
$y_w{}'$	sectional half breadth of waterline
x_b	longitudinal distance of cross-section to centre of gravity, positive forwards
$\overline{OG}$	vertical distance of waterline to centre of gravity, positive upwards
$\overline{BG}$	vertical distance of centre of buoyancy to centre of gravity, positive upwards
∇	volume of displacement
k_{xx}	radius of gyration in air for roll
k_{yy}	radius of gyration in air for pitch
k_{zz}	radius of gyration in air for yaw
r	density of water
g	acceleration of gravity

The solid mass coefficients are given by:

$$M_{1,1} = M = r \cdot \nabla$$
$$M_{2,2} = M = r \cdot \nabla$$
$$M_{3,3} = M = r \cdot \nabla$$
$$M_{4,4} = I_{xx} = k_{xx}{}^{2} \cdot r \cdot \nabla$$
$$M_{5,5} = I_{yy} = k_{yy}{}^{2} \cdot r \cdot \nabla$$
$$M_{6,6} = I_{zz} = k_{zz}{}^{2} \cdot r \cdot \nabla$$

The remaining solid mass coefficients are zero.

The potential mass coefficients are given by:

$$a_{1,1} = \int_L m_{1,1}{}' \cdot dx_b$$
$$a_{1,5} = -\overline{BG} \cdot a_{1,1}$$
$$a_{2,2} = \int_L m_{2,2}{}' \cdot dx_b$$
$$a_{2,4} = \int_L m_{2,4}{}' \cdot dx_b + \overline{OG} \cdot a_{2,2}$$
$$a_{2,6} = \int_L m_{2,2}{}' \cdot x_b \cdot dx_b$$
$$a_{3,3} = \int_L m_{3,3}{}' \cdot dx_b$$
$$a_{3,5} = -\int_L m_{3,3}{}' \cdot x_b \cdot dx_b$$
$$a_{4,4} = \int_L m_{4,4}{}' \cdot dx_b + \overline{OG} \cdot \int_L m_{4,2}{}' \cdot dx_b + \overline{OG} \cdot a_{2,4}$$
$$a_{4,6} = \int_L m_{4,2}{}' \cdot x_b \cdot dx_b + \overline{OG} \cdot a_{2.6}$$
$$a_{5,5} = \int_L m_{3,3}{}' \cdot x_b{}^{2} \cdot dx_b - \overline{BG} \cdot a_{1,5}$$
$$a_{6,6} = \int_L m_{2,2}{}' \cdot x_b{}^{2} \cdot dx_b$$
$$a_{j,i} = a_{i,j}$$

The remaining potential mass coefficients are zero.

The potential mass coefficients are given by:

$$b_{1,1} = \int_L n_{1,1}{}' \cdot dx_b$$
$$b_{1,5} = -\overline{BG} \cdot b_{1,1}$$
$$b_{2,2} = \int_L n_{2,2}{}' \cdot dx_b$$
$$b_{2,4} = \int_L n_{2,4}{}' \cdot dx_b + \overline{OG} \cdot b_{2,2}$$
$$b_{2,6} = \int_L n_{2,2}{}' \cdot x_b \cdot dx_b$$
$$b_{3,3} = \int_L n_{3,3}{}' \cdot dx_b$$
$$b_{3,5} = -\int_L n_{3,3}{}' \cdot x_b \cdot dx_b$$
$$b_{4,4} = \int_L n_{4,4}{}' \cdot dx_b + \overline{OG} \cdot \int_L n_{4,2}{}' \cdot dx_b + \overline{OG} \cdot b_{2,4}$$
$$b_{4,6} = \int_L n_{4,2}{}' \cdot x_b \cdot dx_b + \overline{OG} \cdot b_{2.6}$$
$$b_{5,5} = \int_L n_{3,3}{}' \cdot x_b{}^{2} \cdot dx_b - \overline{BG} \cdot b_{1,5}$$
$$b_{6,6} = \int_L n_{2,2}{}' \cdot x_b{}^{2} \cdot dx_b$$
$$b_{j,i} = b_{i,j}$$

The remaining potential damping coefficients are zero.

The spring coefficients are given by:

$$c_{3,3} = 2 \cdot r \cdot g \cdot \int_L y_w{}' \cdot dx_b$$
$$c_{3,5} = -2 \cdot r \cdot g \cdot \int_L y_w{}' \cdot x_b \cdot dx_b$$
$$c_{4,4} = r \cdot g \cdot \nabla \cdot \overline{GM}$$
$$c_{5,5} = r \cdot g \cdot \nabla \cdot \overline{GM_L}$$
$$c_{j,i} = c_{i,j}$$

The remaining spring coefficients are zero.

The wave loads are given by:

$$F_{w_i} = \int_L F_{w_i}' \cdot dx_b$$

$$\text{with}: F_{w_i}' = m_{1,1}' \cdot \ddot{z}_{w_1}^* + n_{1,1}' \cdot \dot{z}_{w_1}^* + FK_1'$$

$$F_{w_2} = \int_L F_{w_2}' \cdot dx_b$$

$$\text{with}: F_{w_2}' = m_{2,2}' \cdot \ddot{z}_{w_2}^* + n_{2,2}' \cdot \dot{z}_{w_2}^* + FK_2'$$

$$F_{w_3} = \int_L F_{w_3}' \cdot dx_b$$

$$\text{with}: F_{w_3}' = m_{3,3}' \cdot \ddot{z}_{w_3}^* + n_{3,3}' \cdot \dot{z}_{w_3}^* + FK_3'$$

$$F_{w_4} = \int_L F_{w_4}' \cdot dx_b$$

$$\text{with}: F_{w_4}' = m_{4,2}' \cdot \ddot{z}_{w_2}^* + n_{4,2}' \cdot \dot{z}_{w_2}^* + FK_4' + F_{w_2}' \cdot \overline{OG}$$

$$F_{w_5} = \int_L F_{w_5}' \cdot dx_b$$

$$\text{with}: F_{w_5}' = -F_{w_1}' \cdot \overline{BG} - F_{w_3}' \cdot x_b$$

$$F_{w_6} = \int_L F_{w_6}' \cdot dx_b$$

$$\text{with}: F_{w_6}' = F_{w_2}' \cdot x_b$$

These formulations of the hydrodynamic exciting and reaction forces and moments can only be used in the frequency domain, since $a_{i,j}$ and $b_{i,j}$ both depend on the frequency of motion w only and the exciting wave loads have a linear relation with the wave amplitude. In irregular waves the response of the body can be determined by using the superposition principle, so using linear response amplitude operators between motion and wave amplitudes.

In the following figures, an example has been given of the hydrodynamic potential mass and damping and the wave loads for roll in the frequency domain.

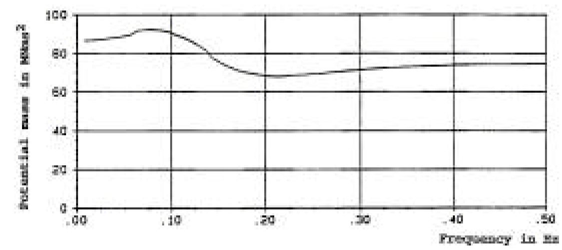

Figure 3-A   Potential Mass of Roll

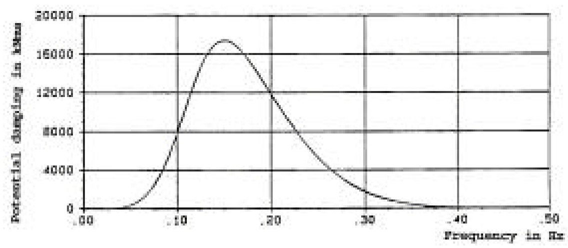

Figure 3-B   Potential Damping of Roll

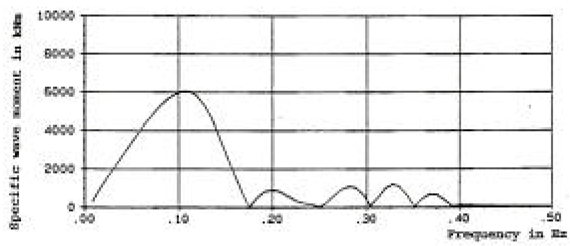

Figure Wave Moment of Roll

4 Time Domain Calculations

As a result of the formulation in the frequency domain, any system influencing the behaviour of the floating body should have a linear relation with the displacement, the velocity and the acceleration of the body. However, in a lot of cases there are several complications which perish this linear assumption, for instance the non- linear viscous damping,

forces and moments due to currents, wind, anchoring, etc.
To include these non-linear effects in the vessel behaviour, it is necessary to formulate the equations of motion in the time domain, which relates instantaneous values of forces, moments and motions.
For the description of the hydromechanic reaction forces and moments, due to time varying ship motions, use has been made of the classic formulation given by Cummins [1] with simple frequency domain solutions of Ogilvie [14].

4.1 Cummins Equations

The floating object is considered to be a linear system, with translational and rotational velocities as input and reaction forces and moments of the surrounding water as output.
The object is supposed to be at rest at time $t = t_0$. Then during a short time Δt an impulsive displacement Δx, with a constant velocity V, is given to the object.
So:

$$\Delta x = V \cdot \Delta t$$

During this impulsive displacement, the water particles will start to move. When assuming that the fluid is rotation-free, a velocity potential Φ, linear proportional to V, can be defined:

$$\Phi = V \cdot \Psi \quad \text{for:} \quad t_0 < t < t_0 + \Delta t$$

in which Ψ is the normalised velocity potential.

After this impulsive displacement Δx, the water particles are still moving. Because the system is assumed to be linear, the motions of the fluid, described by the velocity potential Φ, are proportional to the impulsive displacement Δx.
So:

$$\Phi = c \cdot \Delta x \quad \text{for:} \quad t > t_0 + \Delta t$$

In here, c is a normalised velocity potential.

The impulsive displacement Δx during the period $(t_0, t_0 + \Delta t)$ does not influence the motions of the fluid during this period only, but also further on in time.
This holds that the motions during the period $(t_0, t_0 + \Delta t)$ are influenced also by the motions before this period.

When the object performs an arbitrarily in time varying motion, this motion can be considered as a succession of small impulsive displacements.
Then the resulting total velocity potential $\Phi(t)$ during the period $(t_n, t_n + \Delta t)$ becomes:

$$\Phi(t) = \sum_{j=1}^{6} \left\{ V_{j,n} \cdot \Psi_j + \sum_{k=1}^{n} \left[c_j (t_{n-k}, t_{n-k} + \Delta t) \cdot V_{j,k} \cdot \Delta t \right] \right\}$$

In here:

n	number of time steps
t_n	$= t_0 + n \cdot \Delta t$
t_{n-k}	$= t_0 + (n-k) \cdot \Delta t$
$V_{j,n}$	j^{th} velocity component during period $(t_n, t_n + \Delta t)$
$V_{j,k}$	j^{th} velocity component during period $(t_{n-k}, t_{n-k} + \Delta t)$
Ψ_j	normalised velocity potential caused by a displacement in direction j during period $(t_n, t_n + \Delta t)$
c_j	normalised velocity potential caused by a displacement in direction j during period $(t_{n-k}, t_{n-k} + \Delta t)$

Letting Δt go to zero, yields:

$$\Phi(t) =$$
$$= \sum_{j=1}^{6} \left\{ \dot{x}_j(t) \cdot \Psi_j + \int_{-\infty}^{t} c_j(t - t) \cdot \dot{x}_j(t) \cdot dt \right\}$$

where $\dot{x}_j(t)$ is the j^{th} velocity component at time t.

The pressure in the fluid follows from the linearised equation of Bernoulli:

$$p = -r \cdot \frac{\partial \Phi}{\partial t}$$

An integration of these pressures over the wetted surface S of the floating object provides the expression for the hydrodynamic reaction forces and moments F_i.

With n_i as the generalised directional cosine, F_i becomes:

$$F_i = -\iint_S p \cdot n_i \cdot dS$$
$$= r \cdot \sum_{j=1}^{6} \left\{ \ddot{x}_j(t) \cdot \iint_S \Psi_j \cdot n_i \cdot dS + \right.$$
$$\left. + \int_{-\infty}^{t} \left(\iint_S \frac{\partial c_j(t - t)}{\partial t} \cdot n_i \cdot dS \right) \cdot \dot{x}_j(t) \cdot dt \right\}$$

When defining:

$$A_{i,j} = r \cdot \iint_S \Psi_j \cdot n_i \cdot dS$$
$$B_{i,j}(t) = r \cdot \iint_S \frac{\partial c_j(t - t)}{\partial t} \cdot n_i \cdot dS$$

the hydrodynamic forces and moments become:

$$F_i =$$
$$= \sum_{j=1}^{6} \left\{ A_{i,j} \cdot \ddot{x}_j(t) + \int_{-\infty}^{t} B_{i,j}(t - t) \cdot \dot{x}_j(t) \cdot dt \right\}$$

for $i = 1, 2, \ldots 6$

Together with the linear restoring spring terms $C_{i,j} \cdot x_{i,j}$ and the linear external loads $X_i(t)$, Newton's second law of dynamics provides the linear equations of motion in the time domain:

$$\sum_{j=1}^{6} \left\{ \left(M_{i,j} + A_{i,j} \right) \cdot \ddot{x}_j(t) + \right.$$
$$\left. + \int_{-\infty}^{t} B_{i,j}(t - t) \cdot \dot{x}_j(t) \cdot dt + C_{i,j} \cdot x_j(t) \right\} = X_i(t)$$

for $i = 1, 2, \ldots 6$

in which:

$\ddot{x}_j(t)$	translational or rotational acceleration in direction at time t
$\dot{x}_j(t)$	translational or rotational velocity in direction j at time t
$x_j(t)$	translational or rotational displacement in direction at time t
$M_{i,j}$	solid mass or inertia coefficient
$A_{i,j}$	hydrodynamic mass coefficient
$B_{i,j}$	retardation function
$C_{i,j}$	spring coefficient
$X_i(t)$	external load in direction i at time t

When replacing in the damping part t by $t - t$ and changing the integration boundaries, this part can be written in a more convenient form:

$$\sum_{j=1}^{6} \left\{ \left(M_{i,j} + A_{i,j} \right) \cdot \ddot{x}_j(t) + \right.$$
$$\left. + \int_{0}^{\infty} B_{i,j}(t) \cdot \dot{x}_j(t - t) \cdot dt + C_{i,j} \cdot x_j(t) \right\} = X_i(t)$$

for $i = 1, 2, \ldots 6$

Referring to the classic work on this subject by Cummins [1], these six equations of motion are called here the "Cummins Equations".

4.2 Hydromechanic Coefficients

The linear restoring spring coefficients $C_{3,3}$, $C_{3,5}$, $C_{4,4}$, $C_{5,3}$ and $C_{5,5}$ can be determined easily from the under water geometry and the centre of gravity of the floating object. Generally, the other $C_{i,j}$-values are zero.

To determine $A_{i,j}$ and $B_{i,j}$, the velocity potentials Ψ_j and c_j have to be found, which is very complex.
A much easier method to determine $A_{i,j}$ and $B_{i,j}$ can be obtained by making use of the hydrodynamic mass and damping data found with existing 2-D or 3-D potential theory based computer programs in the frequency domain. Relative simple relations can be found between $A_{i,j}$, $B_{i,j}$ and the calculated data of the hydrodynamic mass and damping in the frequency domain.

For this, the floating object is supposed to carry out a harmonic oscillation x_j with amplitude 1.0 in the direction j:

$$x_j = 1 \cdot \cos(wt)$$

A substitution in the Cummins Equations provides:

$$\begin{aligned} -w^2 \cdot \left(M_{i,j} + A_{i,j}\right) \cdot \cos(wt) \\ -w \cdot \int_0^\infty B_{i,j}(t) \cdot \sin(wt - wt) \cdot dt \\ + C_{i,j} \cdot \cos(wt) \\ = X_i(t) \\ \text{for } i = 1, 2, \ldots 6 \end{aligned}$$

This results into:

$$\begin{aligned} -w^2 \cdot \left\{M_{i,j} + A_{i,j} - \frac{1}{w} \cdot \right. \\ \left. \cdot \int_0^\infty B_{i,j}(t) \cdot \sin(wt) \cdot dt \right\} \cdot \cos(wt) \\ -w \cdot \left\{ \int_0^\infty B_{i,j}(t) \cdot \cos(wt) \cdot dt \right\} \cdot \sin(wt) \\ + \left\{C_{i,j}\right\} \cdot \cos(wt) \\ = X_i(t) \\ \text{for } i = 1, 2, \ldots 6 \end{aligned}$$

In the classic frequency domain description these equations of motion are presented by:

$$\begin{aligned} -w^2 \cdot \left\{M_{i,j} + a_{i,j}(w)\right\} \cdot \cos(wt) \\ -w \cdot \left\{b_{i,j}(w)\right\} \cdot \sin(wt) \\ + \left\{c_{i,j}\right\} \cdot \cos(wt) \\ = X_i(t) \\ \text{for } i = 1, 2, \ldots 6 \end{aligned}$$

In here:

$a_{i,j}(w)$	frequency depending hydrodynamic mass coefficient
$b_{i,j}(w)$	frequency depending hydrodynamic damping coefficient
$c_{i,j}$	restoring spring term coefficient

When comparing these time domain and frequency domain equations - both with linear terms as published by Ogilvie [14] - it is found:

$$a_{i,j}(w) = A_{i,j} - \frac{1}{w} \cdot \int_0^\infty B_{i,j}(t) \cdot \sin(wt) \cdot dt$$

$$b_{i,j}(w) = \int_0^\infty B_{i,j}(t) \cdot \cos(wt) \cdot dt$$

$$c_{i,j} = C_{i,j}$$

After a Fourier re-transformation, the damping term provides the retardation function:

$$B_{i,j}(t) = \frac{2}{p} \cdot \int_0^\infty b_{i,j}(w) \cdot \cos(wt) \cdot dw$$

Then the mass term follows from:

$$A_{i,j} = a_{i,j}(w) + \frac{1}{w} \cdot \int_0^\infty B_{i,j}(t) \cdot \sin(wt) \cdot dt$$

This mass expression is valid for any value of w, so also for $w = \infty$, which provides:

$$A_{i,j} = a_{i,j}(w = \infty)$$

4.3 Addition of Non-Linearities

So far, these equations of motion are linear. But non-linear contributions can be added now to $X_i(t)$ easily.

For instance, non-linear viscous roll damping contributions can be added to X_4 by:

$$\Delta X_4 = -b_{4,4_a}^{(2)} \cdot |\dot{f}| \cdot \dot{f}$$

Also it is possible to include non-linear spring terms, by considering the stability moment as an external load and shifting its contribution to the right hand side of the equation of motion, for instance:

$$C_{4,4} = 0$$

$$\Delta X_4 = -r \cdot g \cdot \nabla \cdot \overline{GN_f}(f) \cdot \sin f$$

in which $\overline{GN_f}(f)$ is the transverse meta-centric height at arbitrarily heeling angles.

4.4 Some Numerical Recipes

Many computer programs fail when calculating $b_{i,j}(w)$ at too high a frequency. This holds that - when determining $B_{i,j}$ - the numerical calculations can be carried out in a limited frequency range $0 \le w \le \Omega$ only:

$$B_{i,j}(t) = \frac{2}{p} \cdot \int_0^\Omega b_{i,j}(w) \cdot \cos(wt) \cdot dw$$

So, a truncation error $\Delta B_{i,j}(t)$ will be introduced:

$$\Delta B_{i,j}(t) = \frac{2}{p} \cdot \int_\Omega^\infty b_{i,j}(w) \cdot \cos(wt) \cdot dw$$

For the uncoupled damping coefficients - so when $i = j$ - this truncation error can be estimated easily.

The relation between the damping coefficient $b_{i,j}(w)$ and the amplitude ratio of the radiated waves and the oscillatory motion $a_{i,i}(w)$ is given by:

$$b_{i,i}(w) = \frac{r \cdot g^2}{w^3} \cdot a_{i,i}^{\,2}(w)$$

From this an approximation can be found for the tail of the damping curve:

$$b_{i,i}(w) = \frac{b_{i,i}}{w^3}$$

The value of $b_{i,i}$ follows from the calculated damping value at the highest frequency used, $w = \Omega$. This holds that a constant amplitude ratio $a_{i,i}(w)$ is supposed here for $w > \Omega$.

Then the truncation error becomes:

$$\Delta B_{i,i}(t) = \frac{b_{i,i} \cdot t^2}{p} \cdot \left\{ \frac{\cos(\Omega t)}{(\Omega t)^2} - \frac{\sin(\Omega t)}{\Omega t} + g + \ln(\Omega t) + \sum_{n=1}^{\infty} \frac{(-1)^n \cdot (\Omega t)^{2n}}{2n \cdot (2n)!} \right\}$$

in which $g = 0.577215...$ (Euler constant)

Studies carried out in the past have showed that in case of a sufficient high value of Ω the contribution of $\Delta B_{i,i}$ into $B_{i,i}$ is often small. The potential damping calculations were based on numerical routines as used in computer program SEAWAY [8,9]. In this program special attention has been paid to the potential calculations at very high frequencies. For normal merchant ships are 5 radians per second, which can be reached by the routines in SEAWAY, a fairly good value for the maximum frequency Ω.
Thus, the retardation function is approximated by the numerical solution of the integral:

$$B_{i,j}(t) = \frac{2}{p} \cdot \int_0^{\Omega} b_{i,j}(w) \cdot \cos(wt) \cdot dt$$

The damping curve has to be calculated at N_w constant frequency intervals Δw, so:

$$N_w \cdot \Delta w = \Omega .$$

When calculating here the retardation functions it assumed that at each frequency interval the damping curve is linearly increasing or decreasing; see Figure 4.

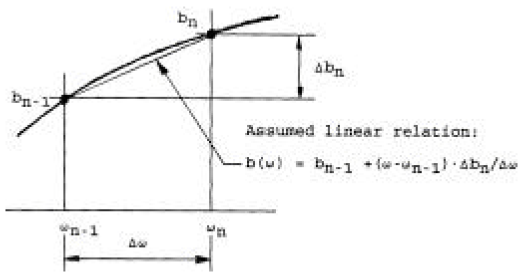

Figure 4 Integration of Damping Curve

Now the contribution of this interval into $B_{i,j}$ can be calculated analytically. This holds that – caused by a large w or a large t - the influence of a strongly fluctuating $\cos(wt)$ at this interval can be taken into account.

Then the numerical integration, at constant frequency intervals Δw, is given by:

$$B_{i,j}(t) = \frac{2}{p \cdot t^2} \cdot \sum_{n=1}^{N_w} \left\{ \frac{\Delta b_n}{\Delta w} \cdot \left[\cos(w_n t) - \cos(w_{n-1} t) \right] \right\} + \frac{2}{p \cdot t} \cdot b_{N_w} \cdot \sin(w_{N_w} t)$$

in which:

$$\Delta w_n = w_n - w_{n-1} = \Delta w$$
$$\Delta b_n = b_n - b_{n-1}$$

For $t = 0$, the value of the retardation function can be derived simply from the integral of the damping:

$$B_{i,j}(t = 0) = \frac{2}{p} \cdot \int_0^{\Omega} b_{i,j}(w) \cdot dw$$

Because the potential damping is zero for $w = 0$, the expression for the damping term leads for $w = 0$, so $\cos(wt) = 1$, into the following requirement for the retardation functions:

$$\int_0^{\infty} B_{i,j}(t) \cdot dt = 0$$

In the equations of motion, the retardation function multiplied with the velocity should be integrated over an infinite time:

$$\int_0^{\infty} B_{i,j}(t) \cdot \dot{x}_j(t - t) \cdot dt$$

However, after a certain time $t = \Gamma_{i,j}$, the fluctuating values of the integral have reached already a very small value.

A useful limit value for the corresponding integration time can be found with:

$$\Gamma_{i,j} = 2 \cdot \sqrt{\frac{\sum_{n=1}^{N_w} |\Delta b_n|}{p \cdot \Delta w \cdot e \cdot B_{i,j}(0)}}$$

with: $e \approx 0.010$

So, the Cummins Equations, which are still linear here, are given by:

$$\sum_{j=1}^{6} \{(M_{i,j} + A_{i,j}) \cdot \ddot{x}_j(t) + \int_0^{\Gamma_{i,j}} B_{i,j}(t) \cdot \dot{x}_j(t-t) \cdot dt + C_{i,j} \cdot x_j(t)\} = X_i(t)$$

for $i = 1, 2, \ldots 6$

The numerical integration can be carried out with the trapezoid rule or with Simpson's rule. Because of a relatively small time step Δt is required to solve the equations of motion numerically, generally the trapezoid rule is sufficient accurate.

The hydrodynamic mass coefficient follows from:

$$A_{i,j} = a_{i,j}(w = \infty)$$

When this mass coefficient is not available for an infinite frequency, it can be calculated from a mass coefficient at a certain frequency $w = \Omega$ and the retardation function:

$$A_{i,j} = a_{i,j}(\Omega) + \frac{1}{\Omega} \cdot \int_0^{\Gamma_{i,j}} B_{i,j}(t) \cdot \sin(\Omega t) \cdot dt$$

With:

$$\Gamma_{i,j} = N_t \cdot \Delta t$$

the numerical solution of this integral can be found in an similar way as for the retardation functions:

$$\int_0^{\Gamma_{i,j}} B_{i,j}(t) \cdot \sin(\Omega t) \cdot dt = \frac{1}{\Omega^2} \cdot \sum_{n=1}^{N_t} \left\{ \frac{\Delta B_n}{\Delta t} \cdot [\sin(\Omega \cdot n \cdot t) - \sin(\Omega \cdot (n-1) \cdot \Delta t)] \right\} + \frac{1}{\Omega} \cdot \{B_{i,j}(t=0) - B_{i,j}(t = t_{N_t}) \cdot \cos(\Omega \cdot N_t \cdot \Delta t)\}$$

in which:

$$\Delta B_n = B_{i,j}(n) - B_{i,j}(n-1)$$

Similar to this, the numerical solution of the frequency depending damping is:

$$b_{i,j}(\Omega) = \int_0^{\Gamma_{i,j}} B_{i,j}(t) \cdot \cos(\Omega t) \cdot dt = \frac{1}{\Omega^2} \cdot \sum_{n=1}^{N_t} \left\{ \frac{\Delta B_n}{\Delta t} \cdot [\cos(\Omega \cdot n \cdot t) - \cos(\Omega \cdot (n-1) \cdot \Delta t)] \right\} + \frac{1}{\Omega} \cdot B_{i,j}(t = t_{N_t}) \cdot \sin(\Omega \cdot N_t \cdot \Delta t)$$

5 Equations of Motion

Integrating the velocities of the ship's centre of gravity can derive the path of the ship in the (x_0, y_0, z_0) system of axes:

$$\dot{x}_0 = \dot{x} \cdot \cos y - \dot{y} \cdot \sin y$$
$$\dot{y}_0 = \dot{x} \cdot \sin y + \dot{y} \cdot \cos y$$
$$\dot{z}_0 = \dot{z}$$
$$\dot{f}_0 = \dot{f}$$
$$\dot{q}_0 = \dot{q}$$
$$\dot{y}_0 = \dot{y}$$

The Euler equations of motion are written in the (x, y, z) system of axes:

$$M\cdot\left(\ddot{x}-\dot{y}\cdot\dot{y}+\dot{z}\cdot\dot{f}\right)=X_h+X_w+X_{ext}$$
$$M\cdot\left(\ddot{y}+\dot{x}\cdot\dot{y}-\dot{z}\cdot\dot{f}\right)=Y_h+Y_w+Y_{ext}$$
$$M\cdot\left(\ddot{z}-\dot{x}\cdot\dot{q}+\dot{y}\cdot\dot{f}\right)=Z_h+Z_w+Z_{ext}$$
$$I_{xx}\cdot\ddot{f}-\left(I_{yy}-I_{zz}\right)\cdot\dot{q}\cdot\dot{y}=K_h+K_w+K_{ext}$$
$$I_{yy}\cdot\ddot{q}-\left(I_{zz}-I_{xx}\right)\cdot\dot{f}\cdot\dot{y}=M_h+M_w+M_{ext}$$
$$I_{zz}\cdot\ddot{y}-\left(I_{xx}-I_{yy}\right)\cdot\dot{f}\cdot\dot{q}=N_h+N_w+N_{ext}$$

with in the right hand sides:

subscript h	linear hydromechanic loads
subscript w	linear wave loads
subscript ext	non-linear hydromechanic loads and (non)linear external loads, caused by wind, currents, anchor lines, cutter, etc.

With the hydromechanic loads as defined before, the equations of motion are defined as given below.

Surge motion:
$$\begin{aligned}&M\cdot\ddot{x}+M\cdot\left(-\dot{y}\cdot\dot{y}+\dot{z}\cdot\dot{q}\right)+\\&A_{1,1}\cdot\ddot{x}+B_{1,1}\cdot\dot{x}+C_{1,1}\cdot x+\\&A_{1,3}\cdot\ddot{z}+B_{1,3}\cdot\dot{z}+C_{1,3}\cdot z+\\&A_{1,5}\cdot\ddot{q}+B_{1,5}\cdot\dot{q}+C_{1,5}\cdot q=X_w+X_{ext}\end{aligned}$$

Sway motion:
$$\begin{aligned}&M\cdot\ddot{y}+M\cdot\left(\dot{x}\cdot\dot{y}-\dot{z}\cdot\dot{f}\right)+\\&A_{2,2}\cdot\ddot{y}+B_{2,2}\cdot\dot{y}+C_{2,2}\cdot y+\\&A_{2,4}\cdot\ddot{f}+B_{2,4}\cdot\dot{f}+C_{2,4}\cdot f+\\&A_{2,6}\cdot\ddot{y}+B_{2,6}\cdot\dot{y}+C_{2,6}\cdot y=Y_w+Y_{ext}\end{aligned}$$

Heave motion:
$$\begin{aligned}&M\cdot\ddot{z}+M\cdot\left(-\dot{x}\cdot\dot{q}+\dot{y}\cdot\dot{f}\right)+\\&A_{3,1}\cdot\ddot{x}+B_{3,1}\cdot\dot{x}+C_{3,1}\cdot x+\\&A_{3,3}\cdot\ddot{z}+B_{3,3}\cdot\dot{z}+C_{3,3}\cdot z+\\&A_{3,5}\cdot\ddot{q}+B_{3,5}\cdot\dot{q}+C_{3,5}\cdot q=Z_w+Z_{ext}\end{aligned}$$

Roll motion:
$$\begin{aligned}&I_{xx}\cdot\ddot{f}-\left(I_{yy}-I_{zz}\right)\cdot\dot{q}\cdot\dot{y}+\\&A_{4,2}\cdot\ddot{y}+B_{4,2}\cdot\dot{y}+C_{4,2}\cdot y+\\&A_{4,4}\cdot\ddot{f}+B_{4,4}\cdot\dot{f}+C_{4,4}\cdot f+\\&A_{4,6}\cdot\ddot{y}+B_{4,6}\cdot\dot{y}+C_{4,6}\cdot y=K_w+K_{ext}\end{aligned}$$

Pitch motion:
$$\begin{aligned}&I_{yy}\cdot\ddot{q}-\left(I_{zz}-I_{xx}\right)\cdot\dot{f}\cdot\dot{y}+\\&A_{5,1}\cdot\ddot{x}+B_{5,1}\cdot\dot{x}+C_{5,1}\cdot x+\\&A_{5,3}\cdot\ddot{z}+B_{5,3}\cdot\dot{z}+C_{5,3}\cdot z+\\&A_{5,5}\cdot\ddot{q}+B_{5,5}\cdot\dot{q}+C_{5,5}\cdot q=M_w+M_{ext}\end{aligned}$$

Yaw motion:
$$\begin{aligned}&I_{zz}\cdot\ddot{y}-\left(I_{zz}-I_{xx}\right)\cdot\dot{f}\cdot\dot{q}+\\&A_{6,2}\cdot\ddot{y}+B_{6,2}\cdot\dot{y}+C_{6,2}\cdot y+\\&A_{6,4}\cdot\ddot{f}+B_{6,4}\cdot\dot{f}+C_{6,4}\cdot f+\\&A_{6,6}\cdot\ddot{y}+B_{6,6}\cdot\dot{y}+C_{6,6}\cdot y=N_w+N_{ext}\end{aligned}$$

Some of the coefficients in these six equations of motion are zero. After omitting these coefficients and ordering the terms, the equations for the accelerations are as follows.

Surge motion:
$$\begin{gathered}\left(M+A_{1,1}\right)\cdot\ddot{x}+A_{1,5}\cdot\ddot{q}\\=\\X_w+X_{ext}\\-B_{1,1}\cdot\dot{x}-B_{1,5}\cdot\dot{q}\\+M\cdot\left(+\dot{y}\cdot\dot{y}-\dot{z}\cdot\dot{q}\right)\end{gathered}$$

Sway motion:
$$\begin{gathered}\left(M+A_{2,2}\right)\cdot\ddot{y}+A_{2,4}\cdot\ddot{f}+A_{2,6}\cdot\ddot{y}\\=\\Y_w+Y_{ext}\\-B_{2,2}\cdot\dot{y}-B_{2,4}\cdot\dot{f}-B_{2,6}\cdot\dot{y}\\+M\cdot\left(-\dot{x}\cdot\dot{y}+\dot{z}\cdot\dot{f}\right)\end{gathered}$$

Heave motion:

$$\begin{gathered}\left(M + A_{3,3}\right)\cdot \ddot{z} + A_{3,5}\cdot \ddot{q} \\ = \\ Z_w + Z_{ext} \\ - B_{3,3}\cdot \dot{z} - C_{3,3}\cdot z - B_{3,5}\cdot \dot{q} - C_{3,5}\cdot q \\ + M\cdot\left(+\dot{x}\cdot\dot{q} - \dot{y}\cdot\dot{f}\right)\end{gathered}$$

Roll motion:

$$\begin{gathered}\left(I_{xx} + A_{4,4}\right)\cdot \ddot{f} + A_{4,2}\cdot \ddot{y} + A_{4,6}\cdot \ddot{y} \\ = \\ K_w + K_{ext} \\ - B_{4,2}\cdot \dot{y} - B_{4,4}\cdot \dot{f} - B_{4,6}\cdot \dot{y} - C_{4,4}\cdot f \\ + \left(I_{yy} - I_{zz}\right)\cdot \dot{q}\cdot \dot{y}\end{gathered}$$

Pitch motion:

$$\begin{gathered}\left(I_{yy} + A_{5,5}\right)\cdot \ddot{q} + A_{5,1}\cdot \ddot{x} + A_{5,3}\cdot \ddot{z} \\ = \\ M_w + M_{ext} \\ - B_{5,1}\cdot \dot{x} - B_{5,3}\cdot \dot{z} - C_{5,3}\cdot z - B_{5,5}\cdot \dot{q} - C_{5,5}\cdot q \\ + \left(I_{zz} - I_{xx}\right)\cdot \dot{f}\cdot \dot{y}\end{gathered}$$

Yaw motion:

$$\begin{gathered}\left(I_{zz} + A_{6,6}\right)\cdot \ddot{y} + A_{6,2}\cdot \ddot{y} + A_{6,4}\cdot \ddot{f} \\ = \\ N_w + N_{ext} \\ - B_{6,2}\cdot \dot{y} - B_{6,4}\cdot \dot{f} - B_{6,6}\cdot \dot{y} \\ + \left(I_{xx} - I_{yy}\right)\cdot \dot{f}\cdot \dot{q}\end{gathered}$$

With known coefficients and right hand sides of these equations, the six accelerations can be determined by a numerical method as - for instance - the well-known Runge-Kutta method.
Because of sometimes an extreme high stiffness of the cutter dredge system, Delft Hydraulics has adapted the numerical solution method of these equations in DREDMO for this; see [13].

6 Viscous Damping

Sway and Yaw

The non-linear viscous sway and yaw damping term in the equations of motion for sway and yaw can be approximated by:

$$b_{2,2_v}{}^{(2)}\cdot\left|\dot{y}\right|\cdot\dot{y} \quad \text{and} \quad b_{6,6_v}{}^{(2)}\cdot\left|\dot{y}\right|\cdot\dot{y}$$

with:

$$\begin{aligned} b_{2,2_v}{}^{(2)} &= \frac{1}{2}\cdot r\cdot L\cdot d\cdot C_D \\ b_{6,6_v}{}^{(2)} &= \frac{1}{6}\cdot r\cdot L^3\cdot d\cdot C_D \\ C_D &\approx 1.50 \end{aligned}$$

Roll

The total non-linear roll damping term in the equation of motion for roll can be expressed as:

$$\left(b_{4,4} + b_{4,4_a}{}^{(1)}\right)\cdot\dot{f} + b_{4,4_a}{}^{(2)}\cdot\left|\dot{f}\right|\cdot\dot{f}$$

with:

$b_{4,4}$	linear potential roll damping coefficient
$b_{4,4_a}{}^{(1)}$	linear(ised) additional roll damping coefficient
$b_{4,4_a}{}^{(2)}$	non-linear additional roll damping coefficient

The linear potential roll-damping coefficient $b_{4,4}$ can be determined as described before.
For time domain calculations a linear as well as a non-linear roll-damping coefficient can be used. However, for frequency domain calculations an equivalent linear roll-damping coefficient has to be estimated. This linearised roll-damping coefficient can be found by requiring that an equivalent linear damping dissipate an equal amount of energy as the non-linear damping, so:

$$b_{4,4_a}^{(1)} \cdot \int_0^{T_f} \dot{f} \cdot \dot{f} \cdot dt = b_{4,4_a}^{(2)} \cdot \int_0^{T_f} |\dot{f}| \cdot \dot{f} \cdot \dot{f} \cdot dt$$

Then the equivalent linear additional roll-damping coefficient $b_{4,4_a}^{(1)}$ becomes:

$$b_{4,4_a}^{(1)} = \frac{8}{3 \cdot p} \cdot f_a \cdot w \cdot b_{4,4_a}^{(2)}$$

The additional roll damping coefficients $b_{4,4_a}^{(1)}$ and $b_{4,4_a}^{(2)}$ are mainly caused by viscous effects. Until now it is not possible to determine these additional coefficients in a pure theoretical way. They have to be estimated by free rolling model experiments or by a semi-empirical method, based on theory and a large number of model experiments with systematic varied ship forms. The linear(ised) and the non-linear equations of pure roll motions, used to analyse free rolling model experiments, are presented here. Also, for zero forward ship speed, the algorithms of the empirical method of Ikeda, Himeno and Tanaka [3] are given.

6.1 Experimental Roll Damping

In case of pure free rolling in still water, the linear equation of the roll motion about the centre of gravity G is given by:

$$\left(I_{xx} + a_{4,4}\right) \cdot \ddot{f} + \left(b_{4,4} + b_{4,4_a}\right) \cdot \dot{f} + c_{4,4} \cdot f = 0$$

in which:

$a_{4,4}$	potential mass coefficient
$b_{4,4}$	potential damping coefficient
$b_{4,4_a}$	linear(ised) additional damping coefficient
$c_{4,4}$	restoring term coefficient

This equation can be rewritten as:

$$\ddot{f} + 2 \cdot n \cdot \dot{f} + w_0^2 \cdot f = 0$$

in which $2n = \frac{b_{4,4} + b_{4,4_a}}{I_{xx} + a_{4,4}}$ is the quotient between damping and moment of inertia and $w_0^2 = \frac{c_{4,4}}{I_{xx} + a_{4,4}}$ is the not-damped natural roll frequency squared.

When defining a non-dimensional roll damping coefficient by:

$$k = \frac{n}{w_0}$$

the equation of motion for roll can be rewritten as:

$$\ddot{f} + 2 \cdot k \cdot w_0 \cdot \dot{f} + w_0^2 \cdot f = 0$$

Then, the logarithmic decrement of roll is:

$$n \cdot T_f = k \cdot w_0 \cdot T_f = \ln\left(\frac{f(t)}{f(t+T_f)}\right)$$

Because of the relation $w_f^2 = w_0^2 - n^2$ and the assumption that $n^2 << w_0^2$, it can be written $w_f \approx w_0$.
This leads to:

$$w_0 \cdot T_f \approx w_f \cdot T_f = 2 \cdot p$$

So, the non-dimensional total roll damping is given by:

$$k = \frac{1}{2 \cdot p} \cdot \ln\left(\frac{w(t)}{w(t+T_f)}\right)$$

$$= \left(b_{4,4} + b_{4,4_a}\right) \cdot \frac{w_0}{2 \cdot c_{4,4}}$$

The non-potential part of the total roll-damping coefficient follows from the average value of k by:

$$b_{4,4_a} = k \cdot \frac{2 \cdot c_{4,4}}{w_0} - b_{4,4}$$

When data on free-rolling experiments with a model in still water are available, these k -values can easily been found.

Often the results of these free rolling tests are presented by:

$$\frac{\Delta \bar{f}_a}{\bar{f}_a} \text{ as function of } \bar{f}_a ,$$

with $\bar{f}_a$ as the absolute value of the average of two successive positive or negative maximum roll angles:

$$\bar{f}_a = \left| \frac{f_a(i) + f_a(i+1)}{2} \right|$$

and $\Delta \bar{f}_a$ as the absolute value of the difference of two successive positive or negative maximum roll angles:

$$\Delta \bar{f}_a = \left| f_a(i) - f_a(i+1) \right|$$

Then the total non-dimensional natural frequency becomes:

$$k = \frac{1}{2 \cdot p} \cdot \ln \left(\frac{2 + \frac{\Delta \bar{f}_a}{\bar{f}_a}}{2 - \frac{\Delta \bar{f}_a}{\bar{f}_a}} \right)$$

These experiments deliver no information on the relation with the frequency of oscillation. So, it has to be decided to keep the additional coefficient $b_{4,4_a}$ or the total coefficient $b_{4,4} + b_{4,4_a}$ constant.

The successively found values for k, plotted on base of the average roll amplitude, will often have a non-linear behaviour as illustrated in Figure 5.
For behaviour like this, it will be found:

$$k = k_1 + k_2 \cdot f_a$$

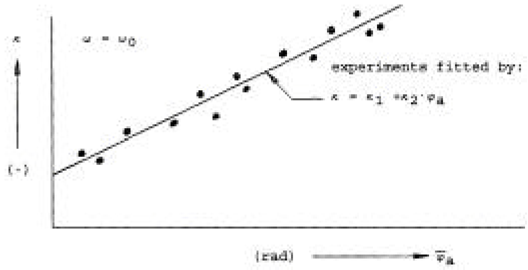

Figure 5 Free Rolling Data

This holds that during frequency domain calculations, the damping term is depending on the solution for the roll amplitude.
For rectangular barges $(LxBxd)$ with the centre of gravity in the water line, it is found by Journée [7]:

$$k_1 \approx 0.0013 \cdot \left(\frac{B}{d} \right)^2$$
$$k_2 \approx 0.50$$

Then the total roll-damping term becomes:

$$\left\{ k \cdot \frac{2 \cdot c_{4,4}}{w_0} \right\} \cdot \dot{f} = \left\{ \left(k_1 + k_2 \cdot f_a \right) \cdot \frac{2 \cdot c_{4,4}}{w_0} \right\} \cdot \dot{f}$$

The linear additional roll-damping coefficient becomes:

$$b_{4,4_a}{}^{(1)} = k_1 \cdot \frac{2 \cdot c_{4,4}}{w_0} - b_{4,4}$$

But for the non-linear additional roll-damping coefficient, a quasi-quadratic damping coefficient is found:

$$b_{4,4_a}{}^{(2)} = k_2 \cdot \frac{2 \cdot c_{4,4}}{w_0} \cdot \frac{f_a}{|\dot{f}|}$$

Because this roll-damping "coefficient" includes $|\dot{f}(w,t)|$ in the denominator, it varies strongly with time.

An equivalent non-linear damping term can be found by requiring that the equivalent quadratic damping term will dissipate an equal amount of energy as the quasi-quadratic damping term, so:

$$b_{4,4_a}{}^{(2)} \cdot \int_0^{T_f} |\dot{f}| \cdot \dot{f} \cdot \dot{f} \cdot dt =$$

$$= k_2 \cdot \frac{2 \cdot c_{4,4}}{w_0} \cdot f_a \cdot \int_0^{T_f} \dot{f} \cdot \dot{f} \cdot dt$$

Then the equivalent quadratic additional roll-damping coefficient $b_{4,4_a}{}^{(2)}$ becomes:

$$b_{44_a}{}^{(2)} = k_2 \cdot \frac{3 \cdot p}{8} \cdot \frac{2 \cdot c_{4,4}}{w_0{}^2}$$

With this, the roll-damping term based on experimentally determined k -values, as given in Figure 5, becomes:

$$\left(b_{4,4} + b_{44_a}{}^{(1)}\right) \cdot \dot{f} + b_{44_a}{}^{(2)} \cdot |\dot{f}| \cdot \dot{f} =$$

$$= k_1 \cdot \frac{2 \cdot c_{4,4}}{w_0} \cdot \dot{f} + k_2 \cdot \frac{3 \cdot p}{8} \cdot \frac{2 \cdot c_{4,4}}{w_0{}^2} \cdot |\dot{f}| \cdot \dot{f}$$

So far, pure roll motions with one degree of freedom are considered in the equations of motion. Coupling effects between the roll motion and the other motions are not taken into account. This can be done in an iterative way.

Experimental or empirical values of k_1 and k_2 provide starting values for $b_{4,4_a}{}^{(1)}$ and $b_{4,4_a}{}^{(2)}$. With these coefficients, a free-rolling experiment with all degrees of freedom can be simulated in the time domain. An analyse of this simulated roll motion, as being a linear pure roll motion with one degree of freedom, delivers new values for $b_{4,4_a}{}^{(1)}$ and $b_{4,4_a}{}^{(2)}$. This procedure has to be repeated until a suitable convergence has been reached. An inclusion of the natural frequency w_0 in this iterative procedure provides also a reliable value for the estimated solid mass moment of inertia coefficient I_{xx}.
However, this procedure is not included in DREDMO yet.

6.2 Empirical Roll Damping

Because of the additional part of the roll damping is significantly influenced by the viscosity of the fluid, it is not possible to calculate the total roll damping in a pure theoretical way. Besides this, experiments showed also a non-linear (about quadratic) behaviour of the additional parts of the roll damping.
As mentioned before, the total non-linear roll damping term in the left-hand side of the equation of motion for roll can be expressed as:

$$\left(b_{4,4} + b_{44_a}{}^{(1)}\right) \cdot \dot{f} + b_{44_a}{}^{(2)} \cdot |\dot{f}| \cdot \dot{f}$$

For the estimation of the additional parts of the roll damping, use has been made of work published by Ikeda, Himeno and Tanaka [3]. Their empirical method is called here the “Ikeda Method”.
At zero forward speed, this Ikeda method estimates the following components of the additional roll-damping coefficient of a ship:

$$b_{4,4_a}{}^{(1)} = 0$$

$$b_{4,4_a}{}^{(2)} = b_{4,4_f}{}^{(2)} + b_{4,4_e}{}^{(2)} + b_{4,4_k}{}^{(2)}$$

with:

$b_{4,4_f}{}^{(2)}$ non-linear friction damping

$b_{4,4_e}{}^{(2)}$ non-linear eddy damping

$b_{4,4_k}{}^{(2)}$ non-linear bilge keel damping

Ikeda, Himeno and Tanaka claim fairly good agreements between their prediction method and experimental results.
They conclude that the method can be used safely for ordinary ship forms. But for unusual ship forms, very full ship forms and ships with a large breadth to draught ratio the method should not be always sufficiently accurate.
Even a few cross sections with a large breadth to draught ratio can result in an extremely large eddy-making component of the roll damping. So, always judge the components of this damping.

In the description of the Ikeda method, the nomenclature of Ikeda is maintained here as far as possible:

r	density of water
n	kinematic viscosity of water
g	acceleration of gravity
w	circular roll frequency
f_a	roll amplitude
R_n	Reynolds number
L	length of the ship
B	breadth of the ship
D	average draught of the ship
C_B	block coefficient
S_f	hull surface area
$\overline{OG}$	distance of centre of gravity above still water level
B_s	sectional breadth on the water line
D_s	sectional draught
s_s	sectional area coefficient
H_0	sectional half breadth to draught ratio
a_1	sectional Lewis coefficient
a_3	sectional Lewis coefficient
M_s	sectional Lewis scale factor
r_f	average distance between roll axis and hull surface
h_k	height of the bilge keels
L_k	length of the bilge keels
r_k	distance between roll axis and bilge keel
f_k	correction for increase of flow velocity at the bilge
C_p	pressure coefficient
l_m	lever of the moment
r_b	local radius of the bilge circle

For numerical reasons two restrictions have to be made during the sectional calculations:

- if $s_s > 0.999$ then $s_s = 0.999$
- if $\overline{OG} < -D_s \cdot s_s$ then $\overline{OG} = -D_s \cdot s_s$

6.2.1 Frictional Damping, $b_{4,4_f}{}^{(2)}$

Kato deduced semi-empirical formulas for the frictional roll-damping from experimental results of circular cylinders, wholly immersed in the fluid.
An effective Reynolds number for the roll motion was defined by:

$$R_n = \frac{0.512 \cdot \left(r_f \cdot f_a\right)^2 \cdot w}{n}$$

In here, for ship forms the average distance between the roll axis and the hull surface r_f can be approximated by:

$$r_f = \frac{\left(0.887 + 0.145 \cdot C_B\right) \cdot \dfrac{S_f}{L} + 2.0 \cdot \overline{OG}}{p}$$

with a wetted hull surface area S_f, approximated by:

$$S_f = L \cdot \left(1.70 \cdot D + C_B \cdot B\right)$$

When eliminating the temperature of water, the kinematic viscosity can be expressed in the density of water with the following relation in the kg-m-s system:

- fresh water:

$$n \cdot 10^6 = 1.442 + 0.3924 \cdot (r - 1000) + 0.07424 \cdot (r - 1000)^2 \quad \mathrm{m^2 s}$$

- sea water:

$$n \cdot 10^6 = 1.063 + 0.1039 \cdot (r - 1000) + 0.02602 \cdot (r - 1000)^2 \quad \mathrm{m^2 s}$$

Kato expressed the skin friction coefficient as:

$$C_f = 1.328 \cdot R_n^{-0.500} + 0.014 \cdot R_n^{-0.114}$$

The first part in this expression represents the laminar flow case. The second part has been ignored by Ikeda, but has been included here.

Using this, the non-linear roll-damping coefficient due to skin friction at zero forward speed is expressed as:

$$b_{4,4_f} = \frac{1}{2} \cdot r \cdot r_f^{\,3} \cdot S_f \cdot C_f$$

Ikeda confirmed the use of this formula for the three-dimensional turbulent boundary layer over the hull of an oscillating ellipsoid in roll motion.

6.2.2 Eddy-Making Damping, $b_{4,4_e}^{(2)}$

At zero forward speed the eddy making roll damping for the naked hull is mainly caused by vortices, generated by a two-dimensional separation. From a number of experiments with two-dimensional cylinders it was found that for a naked hull this component of the roll moment is proportional to the roll velocity squared and the roll amplitude. This means that the non-linear roll-damping coefficient does not depend on the period parameter but on the hull form only.

When using a simple form for the pressure distribution on the hull surface it appears that the pressure coefficient C_p is a function of the ratio g of the maximum relative velocity $U_{\max}$ and the mean velocity U_{mean} on the hull surface:

$$g = \frac{U_{\max}}{U_{\mathrm{mean}}}$$

The relation between C_p and g was obtained from experimental roll damping data of two-dimensional models.

These experimental results are fitted by:

$$C_p = 0.435 \cdot e^{-g} - 2.0 \cdot e^{-0.187 g} + 1.50$$

The value of g around a cross-section is approximated by the potential flow theory for a rotating Lewis-form cylinder in an infinite fluid.

An estimation of the sectional maximum distance between the roll axis and the hull surface, $r_{\max}$, has to be made.

Values of $r_{\max}(y)$ have to be calculated for:

$$y = y_1 = 0.0$$

and:

$$y = y_2 = 0.5 \cdot \arccos\left(\frac{a_1 \cdot (1 + a_3)}{4 \cdot a_3}\right)$$

The values of $r_{\max}(y)$ follow from:

$$r_{\max}(y) = M_s \cdot \sqrt{\begin{array}{l}\{(1 + a_1) \cdot \sin(y) - a_3 \cdot \sin(3 \cdot y)\}^2 + \\ \{(1 - a_1) \cdot \cos(y) + a_3 \cdot \cos(3 \cdot y)\}^2\end{array}}$$

With these two results, $r_{\max}$ and y follow from the conditions:

- if $r_{\max}(\psi_1) > r_{\max}(\psi_2)$ then:

$$r_{\max} = r_{\max}(\psi_1) \quad \text{and} \quad \psi = \psi_1$$

- if $r_{\max}(\psi_1) < r_{\max}(\psi_2)$ then:

$$r_{\max} = r_{\max}(\psi_2) \quad \text{and} \quad \psi = \psi_2$$

The relative velocity ratio γ on a cross-section is obtained by:

$$\gamma = \frac{\sqrt{\pi} \cdot f_3}{2 \cdot D_s \cdot \sqrt{H_0 \cdot \left(\sigma_s + \frac{\overline{OG}}{D_s} \right)}} \cdot \left(r_{\max} + \frac{2 \cdot M_s}{H} \cdot \sqrt{a^2 + b^2} \right)$$

with:

$$\begin{aligned} H = {} & 1 + a_1^2 + 9 \cdot a_3^2 \\ & + 2 \cdot a_1 \cdot (1 - 3 \cdot a_3) \cdot \cos(2 \cdot \psi) \\ & - 6 \cdot a_3 \cdot \cos(4 \cdot \psi) \end{aligned}$$

$$\begin{aligned} a = {} & -2 \cdot a_3 \cdot \cos(5 \cdot \psi) \\ & + a_1 \cdot (1 - a_3) \cdot \cos(3 \cdot \psi) \\ & + \left\{ (6 - 3 \cdot a_1) \cdot a_3^2 + \left(a_1^2 - 3 \cdot a_1 \right) \cdot a_3 \right. \\ & \left. \quad + a_1^2 \right\} \cdot \cos(\psi) \end{aligned}$$

$$\begin{aligned} b = {} & -2 \cdot a_3 \cdot \sin(5 \cdot \psi) \\ & + a_1 \cdot (1 - a_3) \cdot \sin(3 \cdot \psi) \\ & + \left\{ (6 + 3 \cdot a_1) \cdot a_3^2 + \left(a_1^2 + 3 \cdot a_1 \right) \cdot a_3 \right. \\ & \left. \quad + a_1^2 \right\} \cdot \sin(\psi) \end{aligned}$$

$$f_3 = 1 + 4 \cdot e^{-1.65 \cdot 10^5 \cdot (1 - \sigma_s)^2}$$

With this a non-linear sectional eddy making damping coefficient for zero forward speed follows from:

$$\begin{aligned} B_E' = {} & \frac{1}{2} \cdot \rho \cdot D_s^4 \cdot \left(\frac{r_{\max}}{D_s} \right)^2 \cdot C_p \cdot \\ & \cdot \left\{ \left(1 - \frac{f_1 \cdot R_b}{D_s} \right) \cdot \left(1 + \frac{\overline{OG}}{D_s} + \left[\frac{-f_1 \cdot r_b}{D_s} \right] \right) \right. \\ & \left. + f_2 \cdot \left(H_0 - \frac{f_1 \cdot R_b}{D_s} \right)^2 \right\} \end{aligned}$$

with:

$$f_1 = 0.5 \cdot \{ 1 + \tanh(20 \cdot \sigma_s - 14.0) \}$$

$$\begin{aligned} f_2 = {} & 0.5 \cdot \{ 1 - \cos(\pi \cdot \sigma_s) \} \\ & - 1.5 \cdot \left\{ 1 - e^{5 - 5 \cdot \sigma_s} \right\} \cdot \sin^2(\pi \cdot \sigma_s) \end{aligned}$$

The term between square brackets $\left[\frac{-f_1 \cdot r_b}{D_s} \right]$ is included in the program listing in the paper of Ikeda et. al. [3], but it does not appear in the formulas given in the paper. After contacting Ikeda, this term has been omitted in SEAWAY and SEAWAY-D.

The approximation of the local radius of the bilge circle R_b is:

- for $R_b < D_s$ and $R_b < B_s / 2$:

$$R_b = 2 \cdot D_s \cdot \sqrt{\frac{H_0 \cdot (\sigma_s - 1)}{\pi - 4}}$$

- for $H_0 > 1$ and $R_b > D_s$:

$$R_b = D_s$$

- for $H_0 < 1$ and $R_b < H_0 \cdot D_s$:

$$R_b = B_s / 2$$

For three-dimensional ship forms, the zero forward speed eddy-making damping coefficient is found by integration over the ship length:

$$b_{4,4_e}{}^{(2)} = \int_L B_E{}' \cdot dx_b$$

6.2.3 Bilge Keel Damping, $b_{4,4_k}{}^{(2)}$

The bilge keel component of the non-linear roll-damping coefficient is divided into two components:

- a component B_N due to the normal force of the bilge keels
- a component B_S due to the pressure an the hull surface, created by the bilge keels.

The normal force component B_N of the bilge keel damping can be deduced from experimental results of oscillating flat plates. The drag coefficient C_D depends on the period parameter or the Keulegan-Carpenter number. Ikeda measured this non-linear drag also by carrying out free rolling experiments with an ellipsoid with and without bilge keels.
This results in a non-linear sectional damping coefficient:

$$B_N{}' = r_k{}^3 \cdot h_k \cdot f_k{}^2 \cdot C_D$$

with:

$$C_D = 22.5 \cdot \frac{h_k}{p \cdot r_k \cdot f_a \cdot f_k} + 2.40$$

$$f_k = 1.0 + 0.3 \cdot e^{-160 \cdot (1.0 - s_s)}$$

The local distance between the roll axis and the bilge keel, r_k, will be determined further on.

Assuming a pressure distribution on the hull caused by the bilge keels, a non-linear sectional roll-damping coefficient can be defined:

$$B_S{}' = \frac{1}{2} \cdot r_k{}^2 \cdot f_k{}^2 \cdot \int_0^{h_k} C_p \cdot l_m \cdot dh$$

Ikeda carried out experiments to measure the pressure on the hull surface created by bilge keels. He found that the coefficient $C_p{}^+$ of the pressure on the front-face of the bilge keel does not depending on the period parameter, while the coefficient $C_p{}^-$ of the pressure on the back-face of the bilge keel and the length of the negative pressure region depend on the period parameter.

Ikeda defines an equivalent length of a constant negative pressure region S_0 over the height of the bilge keels, which is fitted to the following empirical formula:

$$S_0 = 0.30 \cdot p \cdot f_k \cdot r_k \cdot f_a + 1.95 \cdot h_k$$

The pressure coefficient on the front-face of the bilge keel is given by:

$$C_p{}^+ = 1.20$$

The pressure coefficient on the back-face of the bilge keel is given by:

$$C_p{}^- = -22.5 \cdot \frac{h_k}{p \cdot r_k \cdot f_k \cdot f_a} - 1.20$$

The sectional pressure moment is given by:

$$\int_0^{h_k} C_p \cdot l_m \cdot dh = D_s{}^2 \cdot \left(-A \cdot C_p{}^- + B \cdot C_p{}^+ \right)$$

with:

$$A = (m_3 + m_4) \cdot m_8 - m_7{}^2$$

$$B = \frac{m_4{}^3}{3 \cdot (H_0 - 0.215 \cdot m_1)} + \frac{(1 - m_1)^2 \cdot (2 \cdot m_3 - m_2)}{6 \cdot (1 - 0.215 \cdot m_1)} + m_1 \cdot (m_3 \cdot m_5 + m_4 \cdot m_6)$$

while:

$$m_1 = \frac{R_b}{D_s}$$

$$m_2 = \frac{-\overline{OG}}{D_s}$$

$$m_3 = 1.0 - m_1 - m_2$$

$$m_4 = H_0 - m_1$$

$$m_5 = \frac{\left\{ \begin{array}{l} 0.414 \cdot H_0 + 0.0651 \cdot m_1{}^2 \\ -(0.382 \cdot H_0 + 0.0106) \cdot m_1 \end{array} \right\}}{(H_0 - 0.215 \cdot m_1) \cdot (1 - 0.215 \cdot m_1)}$$

$$m_6 = \frac{\left\{ \begin{array}{l} 0.414 \cdot H_0 + 0.0651 \cdot m_1{}^2 \\ -(0.382 + 0.0106 \cdot H_0) \cdot m_1 \end{array} \right\}}{(H_0 - 0.215 \cdot m_1) \cdot (1 - 0.215 \cdot m_1)}$$

For $S_0 > 0.25 \cdot p \cdot R_b$:

$$m_7 = \frac{S_0}{D_s} - 0.25 \cdot p \cdot m_1$$

$$m_8 = m_7 + 0.414 \cdot m_1$$

For $S_0 < 0.25 \cdot p \cdot R_b$:

$$m_7 = 0.0$$

$$m_8 = m_7 + 0.414 \cdot m_1 \cdot \left\{ 1 - \cos\left(\frac{S_0}{R_b} \right) \right\}$$

The approximation of the local radius of the bilge circle, R_b, is given before.

The approximation of the local distance between the roll axis and the bilge keel, r_k, is given as:

$$r_k = D_s \cdot \sqrt{ \left(H_0 - 0.293 \cdot \frac{R_b}{D_s} \right)^2 + \left(1.0 + \frac{\overline{OG}}{D_s} - 0.293 \cdot \frac{R_b}{D_s} \right)^2 }$$

The total bilge keel damping coefficient can be obtained now by integrating the sum of the sectional roll damping coefficients $B_N{}'$ and $B_S{}'$ over the length of the bilge keels:

$$b_{4,4_k}{}^{(2)} = \int_{L_k} \left(B_N{}' + B_S{}' \right) \cdot dx_b$$

7 Comparative Simulations

As far as ship motions are concerned, SEAWAY-T is an equivalent of DREDMO. To check the calculation routines for the time domain, as used in the pre-processing program SEAWAY-D and in the time domain program SEAWAY-T, comparisons have been made with the results of the frequency domain program SEAWAY [8] for a number of ship types.
An example of the results of these validations is given here for the S-175 containership design in deep water, as has been used in the Manual of SEAWAY too, with principal dimensions as given below.

Length between perp., L_{pp}	175.00 m
Breadth, B	25.40 m
Amidships draught, d_m	9.50 m
Trim by stern, t	0.00 m
Block coefficient, C_B	0.57
Metacentric height, $\overline{GM}$	0.98 m
Longitudinal CoB, L_{CoB} / L_{pp}	-1.42 %
Radius of inertia, k_{xx} / L_{pp}	0.33
Radius of inertia, k_{yy} / L_{pp}	0.24

Radius of inertia, k_{zz} / L_{pp} 0.24

Height of bilge keels, h_k 0.45 m

Length of bilge keels, L_k 43.75 m

The body plan of this container ship design is given in the Figure 6.

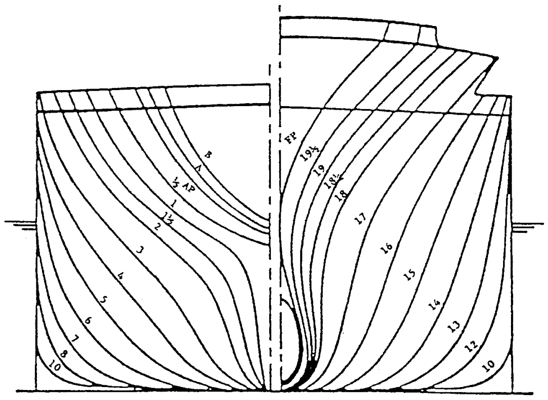

Figure 6 Body Plan S-175 Ship

This S-175 containership design had been subject of several computer and experimental studies, co-ordinated by the Shipbuilding Research Association of Japan and the Seakeeping Committee of the International Towing Tank Conference [4,5]. Results of these studies have been used continuously for validating program SEAWAY after each modification during its development.

For this ship, the motions have been calculated in the frequency domain and in the time domain at zero forward ship speed.

Additional data, used during the time domain simulations, are:

- maximum frequency of damping curves: $\Omega = 5.00$ rad/s
- frequency interval: $\Delta w = 0.05$ rad/s
- maximum time in retardation functions: $\Gamma_{i,j} = 50.00$ s
- time interval: $\Delta t = 0.25$ s

The potential coefficients and the frequency characteristics of the wave loads at zero forward speed, calculated by SEAWAY-D, have been input in SEAWAY-T and the calculations have been carried out for a regular wave amplitude of 1.0 meter. Extra attention has been paid here to the roll motions. In both calculations, the viscous roll damping has been estimated with the Ikeda method. In the frequency domain, the results are linearised for this wave amplitude of 1.0 meter. Because of the relatively small roll-damping at zero forward speed, in the natural frequency region the initial conditions of the wave loads will occur unstable roll motions in the time domain. Then, a long simulation time is required to obtain stable motions.

The agreements between the amplitudes and the phase lags of the six basic motions, calculated both in the frequency domain and in the time domain, are remarkably good. Some comparative results of the six motion amplitudes of the S-175 containership design are given in Table I.

Wave freq. (rad/s)	Motions calculated by SEAWAY and SEAWAY-T respectively						
	x_a (m)	y_a (m)	z_a (m)	ϕ_a (deg)	θ_a (deg)	ψ_a (deg)	PROGRAM
0.2	0.843	0.485	0.992	0.173	0.204	0.088	SEAWAY
	0.82	0.48	0.99	0.18	0.20	0.08	SEAWAY-T
0.3	0.795	0.451	0.956	1.128	0.456	0.195	SEAWAY
	0.78	0.45	0.95	1.13	0.46	0.20	SEAWAY-T
0.4	0.700	0.397	0.860	1.692	0.772	0.282	SEAWAY
	0.69	0.40	0.86	1.71	0.77	0.28	SEAWAY-T
0.5	0.546	0.293	0.671	0.998	1.071	0.357	SEAWAY
	0.54	0.30	0.67	1.01	1.07	0.35	SEAWAY-T
0.6	0.344	0.161	0.385	0.837	1.191	0.338	SEAWAY
	0.34	0.16	0.38	0.85	1.20	0.34	SEAWAY-T
0.7	0.140	0.056	0.173	0.558	0.937	0.211	SEAWAY
	0.14	0.06	0.17	0.56	0.94	0.21	SEAWAY-T
0.8	0.005	0.057	0.395	0.150	0.309	0.039	SEAWAY
	0.00	0.06	0.40	0.15	0.31	0.04	SEAWAY-T
0.9	0.038	0.035	0.385	0.376	0.392	0.069	SEAWAY
	0.04	0.04	0.39	0.38	0.39	0.07	SEAWAY-T
1.0	0.016	0.014	0.101	0.408	0.299	0.046	SEAWAY
	0.02	0.01	0.10	0.41	0.30	0.05	SEAWAY-T
1.1	0.009	0.010	0.027	0.137	0.031	0.017	SEAWAY
	0.01	0.01	0.03	0.14	0.03	0.02	SEAWAY-T
1.2	0.005	0.005	0.002	0.064	0.052	0.014	SEAWAY
	0.01	0.01	0.00	0.06	0.05	0.01	SEAWAY-T
1.3	0.004	0.002	0.010	0.031	0.023	0.010	SEAWAY
	0.00	0.00	0.01	0.03	0.02	0.01	SEAWAY-T
1.4	0.004	0.002	0.005	0.030	0.015	0.003	SEAWAY
	0.00	0.00	0.00	0.03	0.02	0.00	SEAWAY-T
1.5	0.002	0.001	0.003	0.016	0.016	0.003	SEAWAY
	0.00	0.00	0.00	0.02	0.02	0.00	SEAWAY-T

Table I Comparison of Computations for S-175 Ship

Comparisons for a rectangular barge (100 x 20 x 4 meter), with hoppers (25 x 14 meter) fore and aft, are given in Table II for the natural roll frequency region. The experimental roll damping data were input here.

Wave freq. (rad/s)	Motions calculated by SEAWAY and SEAWAY-T respectively						PROGRAM
	x_a (m)	y_a (m)	z_a (m)	ϕ_a (deg)	θ_a (deg)	ψ_a (deg)	
0.3	0.818	0.451	0.980	0.501	0.449	0.223	SEAWAY
	0.81	0.45	0.97	0.50	0.45	0.22	SEAWAY-T
0.4	0.767	0.451	0.937	1.644	0.774	0.375	SEAWAY
	0.76	0.45	0.93	1.65	0.77	0.38	SEAWAY-T
0.5	0.679	0.415	0.846	0.665	1.131	0.530	SEAWAY
	0.67	0.42	0.84	0.67	1.13	0.53	SEAWAY-T

Table II Comparison of Computations for a Barge with Hoppers

Based on these and a lot of other comparisons between the time domain and the frequency domain approaches for linear systems, it may be concluded that SEAWAY-D + SEAWAY-T has an equal accuracy as the frequency domain predictions of these linear motions by the parent program SEAWAY.
This conclusion holds that the pre-processing program SEAWAY-D provides reliable results.

8 Conclusions and Remarks

This new release of SEAWAY-D includes the use of local twin-hull cross-sections, the N-Parameter Close-Fit Conformal Mapping Method and (non)linear viscous roll damping coefficients. Special attention has been paid to longitudinal jumps in the cross sections and fully submerged cross-sections. Also, improved definitions of the hydrodynamic potential masses at an infinite frequency and the wave loads have been added.

Based on a lot of comparisons, made between the time domain and the frequency domain approaches for linear(ised) systems, it may be concluded that the computer codes SEAWAY-D and SEAWAY-T have an equal accuracy as the frequency domain predictions of these linear(ised) motions by the parent code SEAWAY.

This conclusion holds that the pre-processing program SEAWAY-D delivers reliable results. It is advised however to carry out a similar validation study with the computer codes SEAWAY-D and DREDMO too.

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