National Conference on Numerical
Methods for Transient and Coupled
Problems, 9-13 July 1984, Venice, Italy.

Reprinted: 20-11-2000
Website: www.shipmotions.nl

Rapport 0661-P, 1984,
Delft University of Technology,
Ship Hydromechanics Laboratory,
Mekelweg 2, 2628 CD Delft,
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Experimental and Numerical Simulations of Sloshing Behaviour in Liquid Cargo Tanks and its Effect on Ship Motions

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Summary

A two-dimensional finite-difference transient solution has been adapted for the prediction of liquid motions and induced pressures in partially filled ship tanks. Integration of the pressures around the tank walls yield the overall forces and moments transmitted by the liquid onto the tank structure and consequently to the ship. The liquid induced moment is coupled to an equation describing the ship rolling motion and this coupled equation now provides the excitation for the sloshing computer program. Experiments have been conducted on scaled tanks and the measured pressures and moments are compared with the numerical predictions. Further experiments on a ship model, which incorporated partially filled liquid tanks, provide data for comparisons with the coupled sloshing and ship motions computer program.

1 Introduction

Liquid cargo carried in partially filled tanks responds to ship motions and when the excitation is near the natural period of the liquid cargo, violent waves can form which impart dynamic and impact pressures to the tank walls and ceiling. Sloshing loads can be severe enough to cause structural damage; e.g. see ref. [1]. It is therefore necessary to predict the sloshing response at the design stage. It must be pointed out however that realistic predictions are made particularly difficult by the non-linear nature of the phenomenon and by the large number of parameters affecting it, such as tank geometry, liquid-fill height, period and amplitude of excitation and position of the centre of rotation. Lloyd's Register of Shipping adopted in the past an analytical model based on a non-linear theory for shallow-fill cases [2] while for high-fills (i.e. ratio of liquid height to characteristic length greater than 0.2) the solution is obtained from linear potential flow theory [3]. The tank is assumed two-dimensional, with no internal obstructions and no ceiling. Because of the limitations of this solution in the beginning of 1982 a re-examination of the problem was, initiated. It was then decided to employ a numerical rather than analytical method to ensure the generality of modelling tank shape, while the requirement to model the tank ceiling indicated that a transient solution was needed in preference to a steady state one. A literature survey [4, 5] soon indicated the most promising area of recent development. Navickas, et. al. [6] applied the SOLA-SURF variation of the MAC method [7, 8] to the sloshing problem. Navickas modelled a two-dimensional prismatic tank with a ceiling in translational oscillation and extended the MAC code to model liquid compressibility during impacts on the ceiling assuming small changes of density. It was reported [6] that comparisons of compressible and incompressible types of pressure with experiments showed qualitative agreement while very good agreement was observed on comparisons of free surface motions. Furthermore the solution was claimed to be numerically stable. A copy of the computer code was acquired and subsequently modified and extended as described in the next section.

In the middle of 1983 a series of sloshing experiments was conducted at the Ship Hydromechanics Laboratory of the Delft University of Technology. The experiments were jointly commissioned by Boelwerf n.v. Shipyard of Belgium and Lloyd's Register of Shipping. These experiments examined the effects of liquid-fill height, amplitude and period of harmonic motion. The tank was excited in roll, in pitch and in rotation about an axis along the tank diagonal. Typical comparisons between experimental and computed pressures and free surface motions are presented in this paper for the pure roll and for the diagonal excitations.

The computed pressures at each time step of the transient solution are integrated around the walls of the tank to produce the overall forces and moment transmitted from the liquid: to the tank and thus to the ship. Such computations for the resulting moment are also compared in this paper with available experimental measurements. The agreement that is found is very satisfactory and naturally suggests interesting extensions of the computation with applications in the design and tuning of passive anti-roll tanks, in the prediction of dynamic forces on internal structural members, and in a study: of coupling between ship motions and sloshing. In order to provide a tool for assessing the severity of this coupling, when synchronism between ship motions and the natural period of the liquid cargo occurs, a simplified model of the coupled problem has been set up, whereby the one degree of freedom roll equation of the ship “drives” the sloshing program which in turn contributes the additional liquid cargo moment to the equation of motion. An extension of this solution to account for other ship motions (e.g. sway and heave) is presently underway. Fina11y, the predictions for the coupled problem are compared with unpublished measurements from experiments commissioned by Boelwerf n.v. Shipyard on a ship model with three cargo tanks partially filled with liquid, in free roll and in beam waves at zero forward speed. Again the comparison is favourable.

2 Theoretical Method

The two-dimensional finite-difference approach, used here, is often referred to as the “Marker and Cell” technique developed originally at Los Alamos [7, 8]. Essentially the Navier-Stokes equations are solved for each cell of the computational mesh in conjunction with the appropriate boundary conditions and ancillary equations. The solution is advanced through time, using a “snapshot” principle, and enables viscous transient fluid flow problems to be treated. Additional boundary conditions are used to introduce free surfaces into a simulation.

Since the intention at Lloyd's Register was to produce a computer code that could cope with a wide variety of tank shapes and analyse the resulting data from many aspects, many modifications and extensions to the source code by Navickas et. al. [6] were introduced

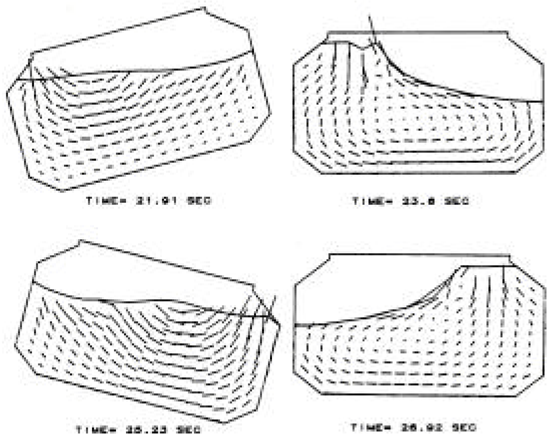

Figure 1 Numerical Simulations at Quarter Period Intervals

Briefly: the formulation of boundary conditions was altered and as a result tanks with chamfered corners, tanks with appreciable internal structure and U-tanks designed for passive roll stabilisation can now be investigated. Also, rotational excitation was introduced since roll is a most relevant motion to liquid cargo sloshing. The forces and moment induced by the liquid on a tank boundary, or on the complete tank, are now computed by integrating pressures along the relevant boundaries. For stable and efficient simulation, a facility has been added to choose automatically the time step by inspecting the velocity distribution and thus choosing a time increment accordingly. A possible building up of numerical errors has been avoided by satisfying the continuity condition before and after updating the free surface. Computer graphics code has been introduced (e.g. see Figure 1), and finally it should be mentioned that the theory developed by Navickas et. al. [6] to handle free surface impacts on the tank ceiling has been rationalised and improved. This enhancement has yet to be fully tested and could be the subject of a future paper.

As a first stage of development for a numerical model of, coupled ship motions and liquid sloshing, the ship is assumed to respond in roll only.

The equation of motion in the absence of internal liquid (frozen condition say) is:

$$I\ddot{f} + N_1\dot{f} + N_3\dot{f}^3 + R_1 f + R_3 f^3 = K_w \sin w_w t \tag{1}$$

where f is the roll angle and a dot denotes a time derivative. I, N_1, N_3, R_1 and R_3 are the inertia of the ship and added inertia, linear and non-linear damping and restoring force coefficients respectively. K_w represents the wave excitation at frequency w and the subscript w denotes the forcing wave. All coefficients, except the, restoring force ones, are dependent on the frequency of excitation and can be obtained by hydrodynamic analysis.

In, the simplified study presented here the coefficients were which was obtained by the, assumed to be constant, except K_w following simple expression from ref. [9]:

$$\begin{aligned} K_w &= R\cdot a\cdot e^{\frac{-w^2 d}{2g}} \\ &= R\cdot\left(\frac{2p}{l}z\right)\cdot e^{\frac{-w^2 d}{2g}} \end{aligned} \tag{2}$$

where d is the ship’s draught while a, z, and l are the slope, amplitude and length of the wave respectively.

When the ship carries liquid cargo, the equation of motion takes the form:

$$\begin{aligned} &(I + I_{fc})\ddot{f} + N_1\dot{f} + N_3\dot{f}^3 \\ &+ \left(R_1 - W\cdot\overline{QG}\right)f + R_3 f^3 = K_w \sin w_w t + C \end{aligned} \tag{3}$$

where I_{fc}, W and $\overline{QG}$ are the cargo's moment of inertia, weight and vertical distance from its centroid to the ship's centre of gravity respectively, all calculated statically.

The term $C(\ddot{f},\dot{f},f)$ is the sloshing induced moment, computed at every time step by the program described earlier, and multiplied by the total length of partially filled tanks. The solution to this equation yields the values of angular displacement, velocity and acceleration, which are used to excite the sloshing analysis in the subsequent time-step. The time phase between the resulting motion and the computed liquid induced moment determines the portions of the moment that should be distributed between the two dynamic and one static terms in the equation of motion. The predicted response in the liquid cargo condition is dependent on the position of the assumed centre of rotation when employing the "artificial" model of the one-degree of freedom equation. In an attempt to simulate the equivalent centre of rotation of a ship - said to be somewhere between the centre of gravity and the free surface - the wave induced moment in the liquid conditions is set proportional to the restoring force reduced by the free surface correction to the metacentric height (K_w)

3 Experiments and Comparisons with Computations

3.1 Pressures and Free Surface Motions

Figure 2 depicts the transparent model (scale 1:40) of a product carrier prismatic tank used in the experiments. DRUCK POCR-42 miniature semiconductor pressure transducers with a range of 10 psid were employed. The natural frequency of the, pressure transducers was lowered by mounting their housing on a steel mass, elastically glued to the outer side of the tank.

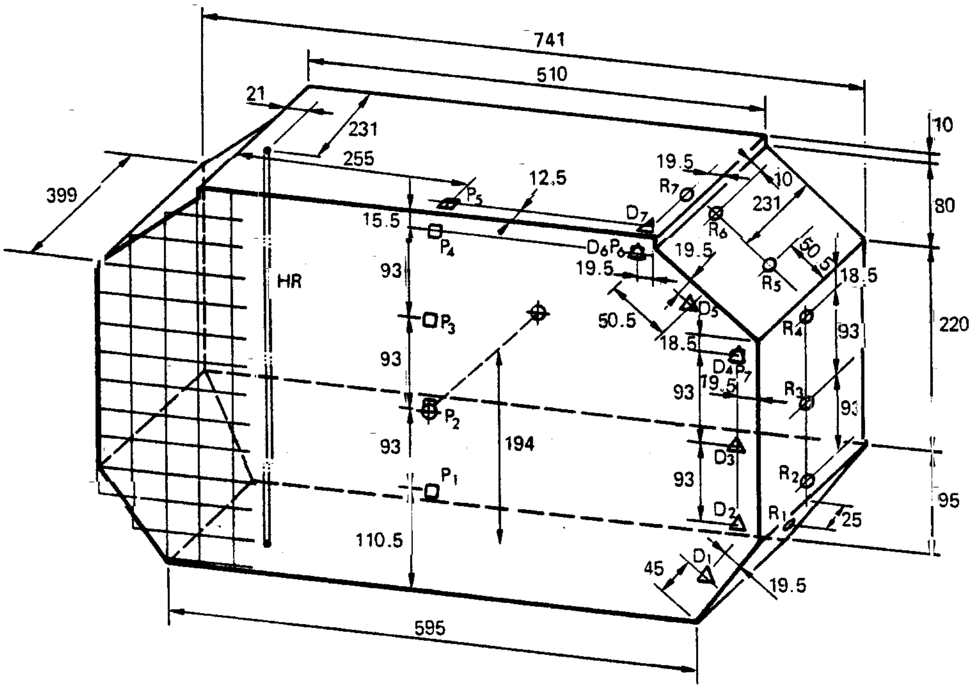

Figure 2 Model Tank, Pressure Transducers, Wave Height Recorder and Part of Computational Grid (in mm)

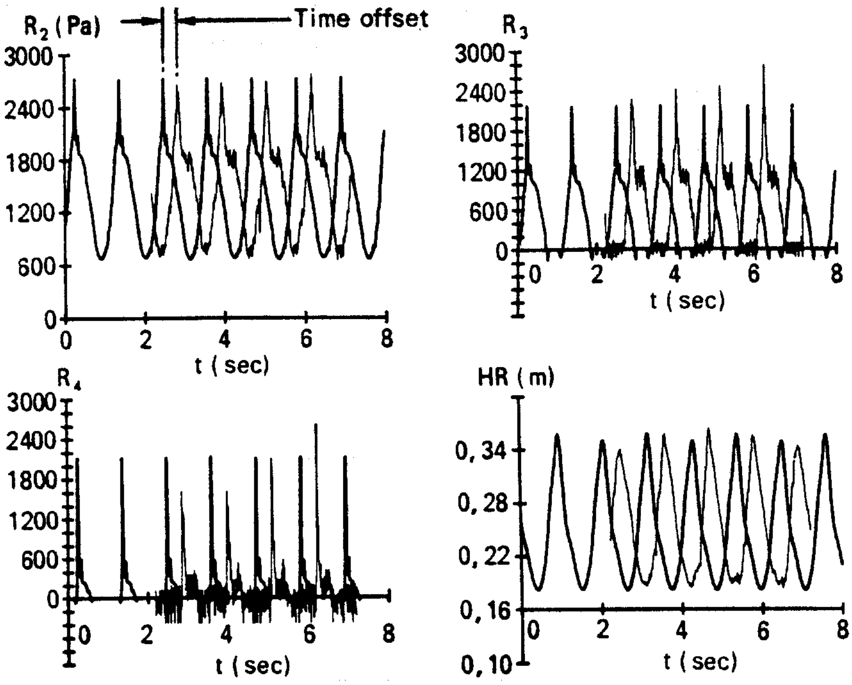

Figure 3 Experimental and Computed Pressures in Pascals at Transducers R2, R3, R4 and Free Surface Height: Roll, $h/D = 0.61$, $T = 1.112$ sec., $f = 0.10$ rad.

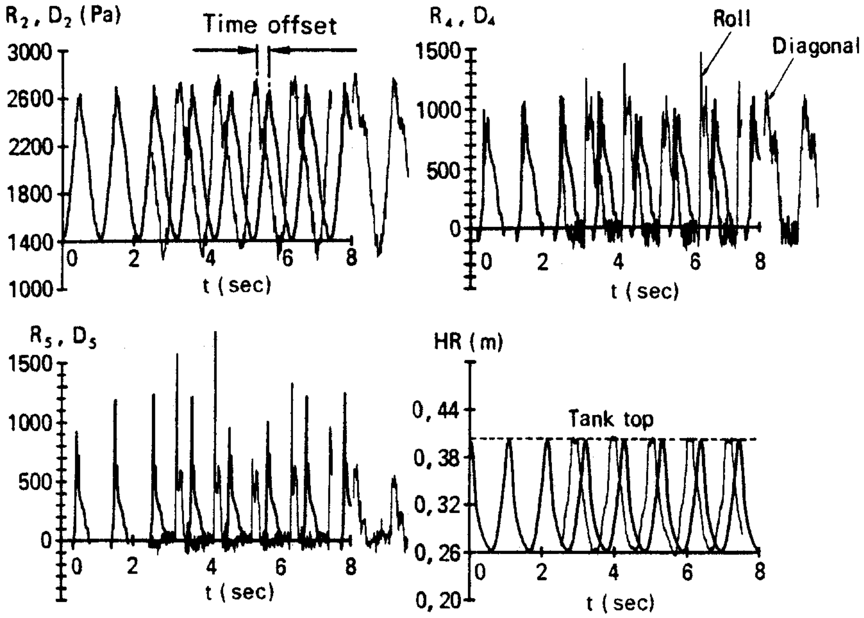

Figure 4 Experimental and Computed Pressures in Pascals at Transducers R2, D2, R4, D4, R5, D5 and Free Surface Height: Roll and Diagonal, $h/D = 0.75$, $T = 1.057$ sec., $f = 0.10$ rad.

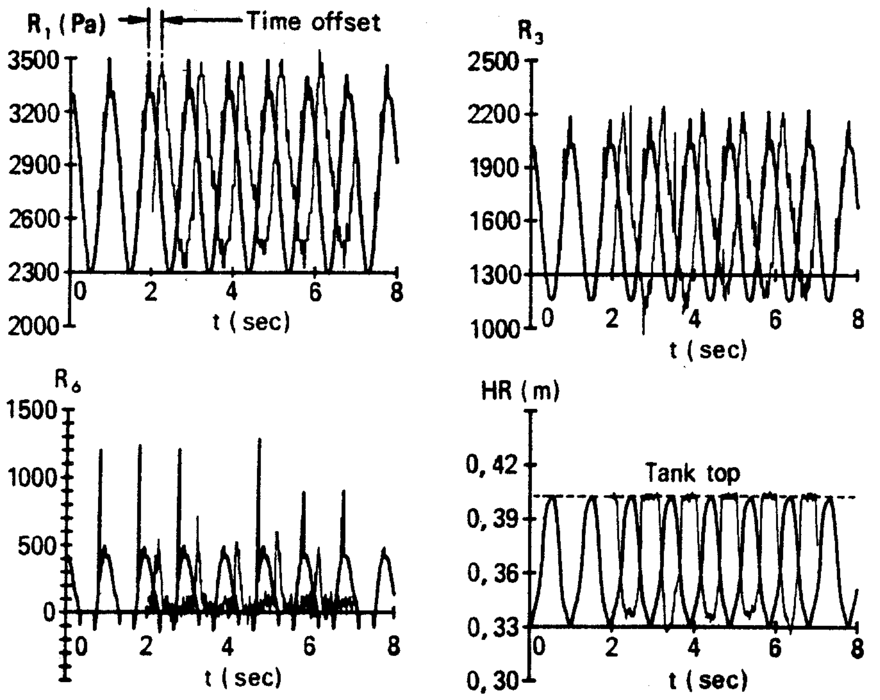

Figure 5 Experimental and Computed Pressures in Pascals at Transducers R1, R3, R6 and Free Surface Height: Roll, $h/D = 0.90$, $T = 0.970$ sec., $f = 0.10$ rad.

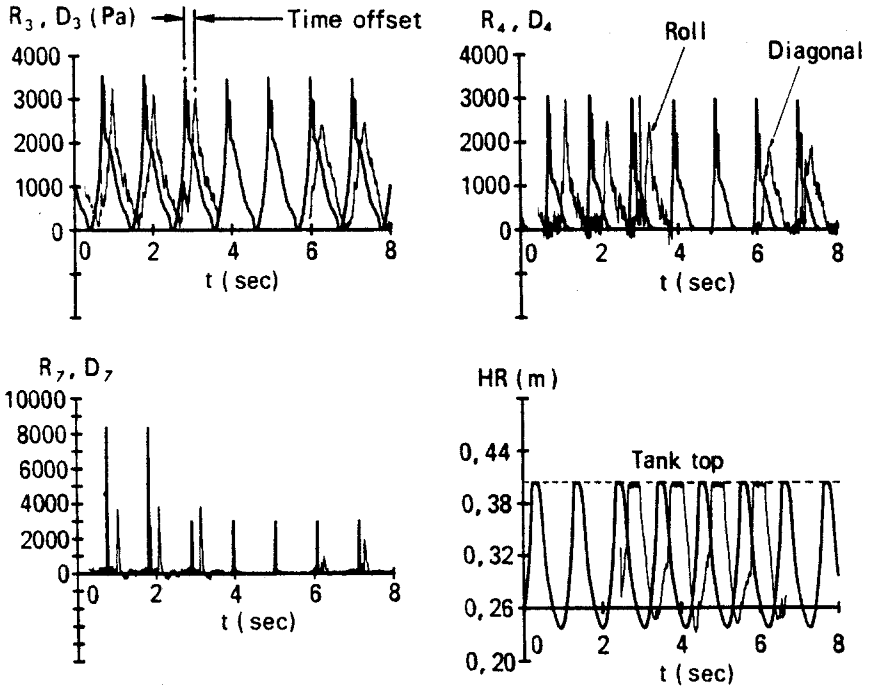

Figure 6 Experimental and Computed Pressures in Pascals at Transducers R2, D3, R4, D4, R7, D7 and Free Surface Height: Roll and Diagonal, $h/D = 0.75$, $T = 1.056$ sec., $f = 0.25$ rad.

A conductive wave probe was used to measure as shown on Figure 2. Harmonic oscillations were carried out with an oscillator mechanism developed at the Delft University of Technology and adapted to the particular requirements of the tests. Three types of oscillations (roll, pitch and rotations about a diagonal axis) were performed, for a range of forcing periods, for six filling levels and in some cases for three different amplitudes of oscillation.

The comparisons shown in this paper are between the incompressible mode computation (Figures 3 to 6, thick lines) and the experimental measurements (thinner lines). Predicted and measured quantities are offset in the time scale for ease of comparison. All time-history traces are plotted by the computer directly and all the cases shown correspond to resonance.

Figures 3 to 5 show comparisons of pressure and free surface motions at different fill levels for the tank rolling with amplitude of 0.1 radians.

Figures 4 and 6 are comparisons at two different amplitudes of excitation for the rolling tank at $h/D = 0.75$, where h is the liquid height and D is the tank's depth. The same two figures also show the recorded pressures from the diagonal excitation tests at the same forcing periods as with the pure roll tests.

Originally it was expected that the diagonal excitation would result in a magnification of motions and pressures as the liquid rushes to the tank corners. However, on these diagonal tests it was observed that when the forcing period was away from either the natural period of roll or of pitch there was very little liquid motion. When the forcing period was in the neighbourhood of either natural periods, then the liquid motion was confined to the plane of motion whose period was excited. In retrospect this is attributed to the fact that, the tank breadth to length ratio (1.86) results in distinctly different natural periods of the liquid cargo in roll and pitch, and thus each of these modes is excited separately.

3.2 Liquid Induced Moments

The forced harmonic roll experiments designed to measure the moment induced by the liquid motion were performed at the Delft Institute of Technology, using the oscillator mechanism described in section 3.1. The moments were measured by two electronic strain gauge dynamometers, which bring over the roll motion and moment from the oscillator to the platform where the tank was fixed. Friction losses in the supporting axis are minimised by an air cushion bearing. To the output of the dynamometer a harmonic analysis was applied, producing the in- and out-phase components of the measured moment signal. The inertia of the tank and of supporting structure was also measured by tests on the empty tank and was subtracted from the experimental values so that the latter show the effect of liquid only. For direct comparison, the computed time history of moment has been Fourier analysed so that the non-dimensional amplitude ($C = m_a / \left(rgB^3 L \right)$) and phase angle e_t of the first harmonic component of the moment was obtained.

Figure 7 shows comparisons between computation and measurement for a rectangular tank with three different shallow fill heights and a range of excitation periods. The experimental data have been published in ref. [9]. Other comparisons with experimental data from the same source, examining the effect of amplitude of motion and position of centre of rotation showed the same good agreement.

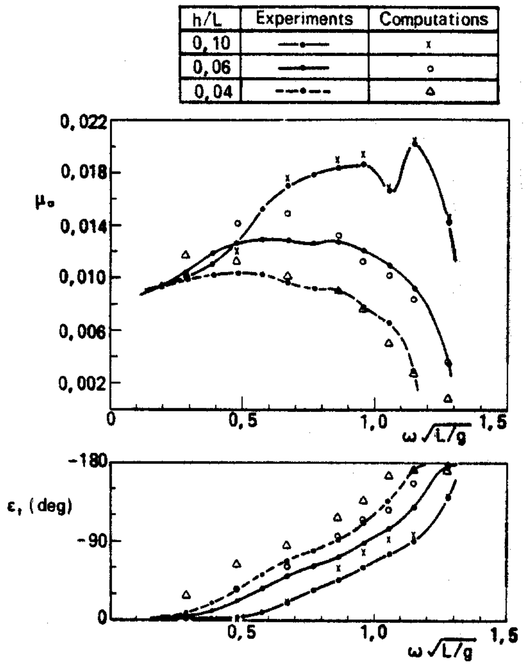

Figure 7 Experimental and Computed Dimensionless Amplitude and Phase Angle of Sloshing Moment on a Rectangular Tank
(Effect of Fill Level, $f = 0.10$ rad.)

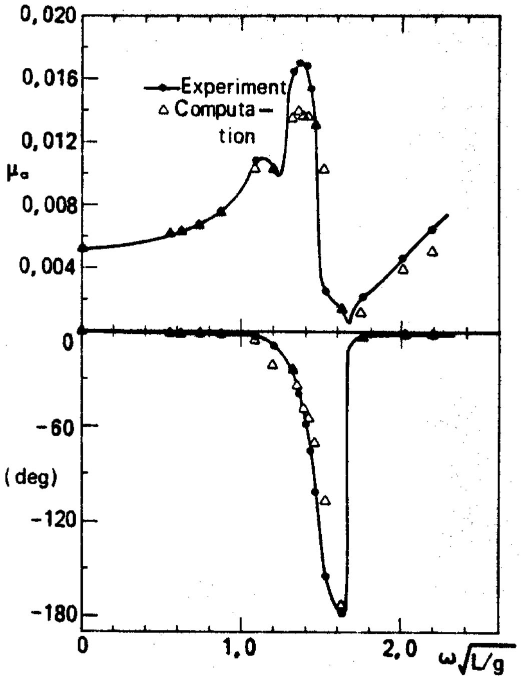

Figure 8 Experimental and Computed Dimensionless Amplitude and Phase Angle of Sloshing Moment on a Rectangular Tank
(Tank of Figure 2 in Roll, $h/D = 0.45$, $f = 0.10$ rad.)

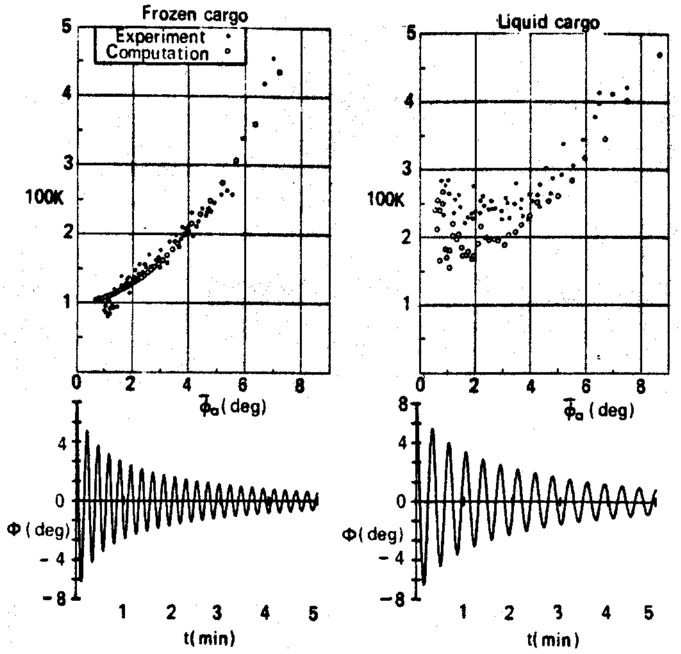

Figure 9 Experimental and Computed Dimensionless Logarithmic Decrement and Computed Angular Displacement in Free Roll from 10^0 with Frozen and Liquid Cargo

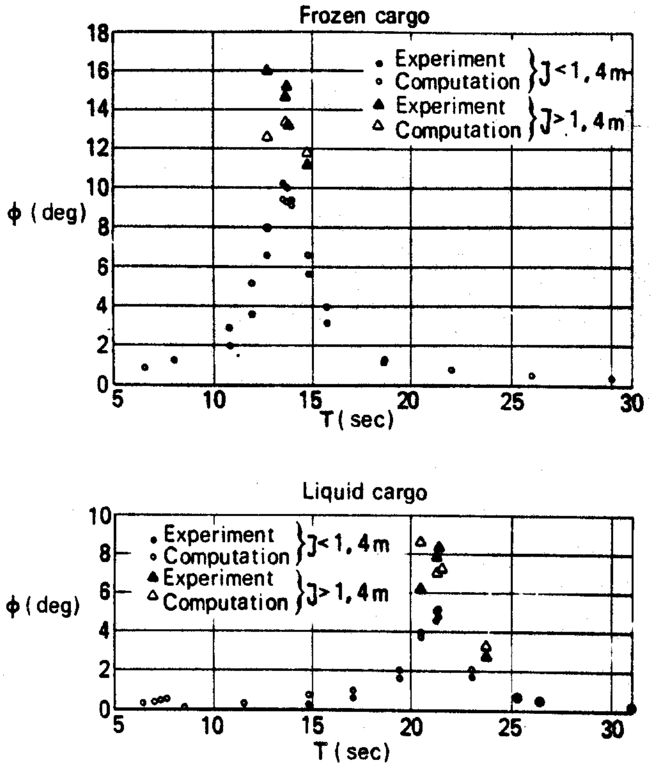

Figure 10 Experimental and Computed Roll Response of Ship Model Incorporating 3 Tanks Filled at $h/D = 0.45$ with Frozen and Liquid Cargo in Beam seas at zero Forward Speed

Figure 8 shows comparisons with unpublished experimental data for the prismatic tank of Figure 2 (this time model to full scale ratio is 1:25). The observed agreement in all cases instilled confidence in the computed moment, prior to developing the coupled motion and sloshing numerical model.

3.3 Coupled Ship Motions and Sloshing

A 1:60 scaled ship model of a product carrier was constructed and was equipped with three cargo tanks. All experiments were carried out at zero forward speed in towing tank 1 of the Delft Ship Hydromechanics Laboratory and the model was placed transversely in the tank to meet beam waves. The waves were produced by a pneumatic wave maker and were measured by a conductive wave probe. During the free rolling tests in still water an inclinometer measured the roll angle time history, while a vertical gyro with potentiometric output was employed in the rolling experiments in beam waves. In these ship model experiments, three conditions were investigated. Each condition included free rolling tests from, an initial roll angle in calm water, and forced roll tests for, a range of wave excitation periods. The three tanks built into the ship model were filled up to 45% of their depth with solid (frozen) cargo in condition I, with liquid cargo in conditions II, and with liquid cargo with three different fill levels in condition III. In the numerical study the free roll data from condition I (frozen cargo) were used to estimate the inertia coefficient of the ship (from its observed natural period) and the linear and non linear damping coefficients (by a least squares fit on the observed logarithmic decrement), whereas the restoring force is obtained statically. In the absence of experimental data a hydrodynamic analysis would provide the necessary coefficients. Assuming constant inertia and damping terms, equations (1) and (3) were used to simulate free and forced roll responses for condition I and II respectively.

Figure 9 shows the experimental and measured free roll response for the two conditions by plots of logarithmic decrement, k, against average roll displacement, where:

$$k = \frac{1}{2p} \ln \left(\frac{f(t)}{f(t+T_n)} \right)$$

$$\bar{f} = \frac{f(t)+f(t+T_n)}{2} \qquad (4)$$

In the frozen cargo condition the agreement between natural periods (13.7 seconds for full-scale ship in both experiment and computation) and between logarithmic decrement distributions is very good, as expected. In the liquid condition the observed natural period is 21.3 seconds and the predicted is 21.4 seconds. The distributions of logarithmic decrement differ now by a uniform shift suggesting that the damping in the computation is underestimated. The distribution of the two sets of values however, is very similar and the computation even predicts the same scatter seen in the experiment.

Comparisons between experiments and computations for the forced roll response in both conditions are shown on Figure 10. The agreement is again very reasonable. The negligible motions predicted away from the ship’s modified natural period and near the liquid cargo natural period, were insufficient to excite any sloshing and related ship motions. This conclusion however applies to the conditions studied, and it is easily conceivable that for other liquid cargo fill levels or for another ship or tank, synchronism could occur with quite different response to that seen here.

Finally, it should be pointed out that although condition III has not been simulated numerically, this could be achieved by simultaneously modelling three tanks with different fill heights.

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5 Notations

B	Breadth of tank
C	Sloshing-induced moment on tank
I	$= I_{ship} + I_{fc}$, Moment of inertia of ship and "frozen" cargo
k	$= \{\ln[f(t)/f(t+T_n)]\}/(2p)$, Roll decrement
K_w	Wave-induced moment
L	Length of tank
N_1, N_3	Hydrodynamic damping coefficients
$\overline{QG}$	Distance from centroid of "frozen" cargo to ship's centre of gravity
R_1, R_3	Restoring moment coefficients
T	Period of forced motion
W	Weight of liquid cargo
d	Ship's draught
g	Gravitational constant
h	Liquid height in tank
t	Time
a	Wave slope amplitude
e_t	Phase angle between sloshing-induced moment and roll displacement, negative for lagging moment
z	Wave height amplitude
l	Length of incident wave
m_a	$= C/(r_{lc} g B^3 L)$, Non-dimensional amplitude of sloshing-induced moment
r	Density
f	$= \Phi \sin wt$, Rotational displacement in roll

$\bar{f}$	$\{f(t)+f(t+T_n)\}/2$, Average roll displacement in free rolling tests
w	Frequency of oscillation
n	Subscript, denoting natural period
w	Subscript, denoting incident wave
fc	Subscript, denoting “frozen” cargo