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Calculation Method for the Behaviour of a Cutter Suction Dredge Operating in Irregular Waves

P.J. Keuning[*] and J.M.J. Journée[**]

[*] **Delft Hydraulics**
[**] **Delft University of Technology**

SUMMARY

The dynamic behaviour of a cutter suction dredge in waves, and especially the reduction of motions and forces, receive increasing attention. For this the availability of numerical tools is essential.
The Delft University of Technology took the initiative to start a research project on improving workability in cooperation with the Delft Hydraulics Laboratory.
One of the elements of this project was the development of a mathematical model for computation of the behaviour of the cutter suction dredge in irregular seas, which is described in this paper.
It is well known that this behaviour is strongly influenced by the non-linear soil reaction on the cutter head. For this reason it is not possible any more to conduct the computation in the frequency domain.
Therefore the motions have to be calculated in the time domain. This makes it necessary to formulate a set of equations, which relates instantaneous values of hydrodynamic forces and motions. Hereto the Cummins approach is used.
A description is given of the developed computer program. Also some computational results are presented. These clearly demonstrate the importance of the incorporation soil behaviour in the mathematical model.

NOMENCLATURE

$a(w)$	added mass
A	cross sectional area
$b(w)$	damping
B	ship's beam
C_D	drag coefficient
C_M	inertia coefficient

C_{kj}	hydrostatic spring coefficient
D_0	cable diameter, unloaded
F_M	inertia force
F_D	drag force
i_s	unit tangent vector to cable element
i_f	unit vector normal to cable in vertical plane
i_q	unit vector normal to i_s and i_f in horizontal plane
l	length
L	ship's length
M_{kj}	mass matrix
N	number
$R_{s,f,q}$	hydrodynamic force per unit length on cable
s	distance along cable
S_{zz}	energy density spectrum
t	time
T	cable tension
v	relative velocity between water and construction
$v_{s,f,q}$	relative water velocity in s, f, q direction
W	net weight per unit length of cable
x_j	degrees of freedom
a_c	current angle of attack
∇	volume of displacement
$z(t)$	wave elevation
z_a	wave amplitude
e	unit elongation
e_j	phase angle
e_{z_i}	wave phase angle
$e_{F_k z_i}$	phase angle between wave and force
n	Poisson ratio
t	time
r	density of water
f	cable angle in vertical plane
q	cable angle in horizontal plane
w	angular velocity

1. INTRODUCTION

During the last decade the operations of cutter suction dredges have shifted from fairly protected waters to near-shore areas. In these areas the dredge is much more exposed to wind-, wave- and current forces, which usually was not anticipated in the design. This results, both, in an increase in downtime and in higher loads on the components of the dredge.

Attempts to improve the design lead to mechanical solutions, such as swell-compensation on the spud carriage and ladder. In very hostile environments, the spud is replaced by a complicated anchoring system, the 'christmas tree', to extend the limit of workability.

As a result of substantial growth of these near shore activities, the behaviour of cutter suction dredges in waves, and especially methods to reduce its motions and forces, are receiving increasing attention from both designers and contractors. For this reason knowledge of the motions of the dredge and the forces on the cutter, spud-pole mooring system, etc. are becoming more and more essential.

Hence the need to develop a mathematical model for the behaviour of a cutter suction dredge, operating in irregular waves, became apparent, in particular to:

- investigating improvements on the design and
- predicting downtime.

The Delft University of Technology took the initiative to start a research project in collaboration with the Delft Hydraulics Laboratory.

This project comprised the following main elements:

1. Review and verification of existing mathematical techniques for computation of wave forces on the dredge. This study was completed in 1979 [1].
2. Development of an analytical description of the soil reaction forces on an oscillating cutter. An extensive research program is being performed at

the Delft University of Technology. The first results are presented at the WODA Congress in 1983 [2].
3. Development of a mathematical model simulating the dynamic behaviour of a cutter suction dredge in waves. This resulted in the DREDMO program, which will be described in detail in this paper.

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2. THE COMPUTATION OF THE BEHAVIOUR OF THE DREDGE

2.1 General

The computation of the behaviour of a floating construction in a seaway is well established in the last decades, using the frequency-domain formulation.
The equation of motion, based on Newton's law of dynamics, is given by:

$$\underline{\underline{M}} \cdot \underline{\ddot{x}} = \underline{F}_{TOT} \tag{1}$$

in which:

$\underline{\underline{M}}$	(6x6) matrix of inertia of the body
$\underline{\ddot{x}}$	(6) acceleration vector of the body in its six degrees of freedom
$\underline{F}_{TOT}$	forces (moments) vector, containing all forces acting on the body.

In case of a cutter suction dredge, the most important contributions to the vector $\underline{F}_{TOT}$ are:

- the wave exciting forces
- the hydrodynamic reaction forces
- the hydrostatic restoring forces
- the external forces due to the mooring system, cutter ladder, wind and current.

For simple harmonic motions the hydrodynamic reaction forces are conventionally expressed in terms of the added mass and damping coefficients $a(w)$ and $b(w)$.
The Equations (1) become:

$$\sum_{j=1}^{6}\left\{-w^2 M_{kj} \cdot x_j\right\} =$$

$$= \sum_{j=1}^{6}\left\{w^2 a_{kj} \cdot x_j - w b_{kj} \cdot x_j - C_{kj} \cdot x_j\right\} +$$

$$+ F_k(w) \cdot \sin(wt + e_k) + F_k(ext)$$

$$k = 1,...6$$

This formulation of the hydrodynamic reaction forces can only be used in the frequency domain, since a_{kj} and b_{kj} depend on the frequency of motion w. The response of the body to irregular waves is determined, using linear response amplitude operators between motion and wave amplitude.
Wichers computed the behaviour of the dredge using this linear approach [3]. Ladder and spud-pole are represented as bodies without inertia forces on the barge.
As a result of the use of the formulation in the frequency domain, any system influencing the behaviour of the floating body, such as spud-pole, ladder and mooring system may only have a linear relation with the displacement, velocity or acceleration of the body. However, for the cutter suction dredge there are several complications, which perish this linearity assumption, the most important one being the soil reaction forces on the cutter, which are known to be strongly non-linear. In order to incorporate these non-linear effects in the dredge behaviour, it is necessary to formulate the equations of motion in the time domain, which relates instantaneous values of forces and motions. For the description of the hydrodynamic reaction forces, due to time varying ship motions, use is made of the formulation as given by Cummins [4]. Considering the floating object to be a linear system between the input (velocity) and output (hydrodynamic reaction force),

the hydrodynamic reaction force F_k becomes:

$$F_k = \sum_{j=1}^{6} \left\{ m_{kj}\ddot{x}_j(t) + \int_{-\infty}^{t} K_{kj}(t-t)\dot{x}_j(t)dt \right\}$$

in which:

m_{kj} added mass tensor

$K_{kj}(t)$ retardation function.

The values of m_{kj} and $K_{kj}(t)$ can be derived using the frequency dependent added mass and damping coefficient $a(w)$ and $b(w)$ [5]:

$$K_{kj}(t) = \frac{2}{p}\int_{0}^{\infty} b_{kj}(w)\cos(wt)dw$$

$$m_{kj} = a_{kj}(w) + \frac{1}{w}\int_{0}^{\infty} K_{kj}(t)\sin(wt)dt$$

(2)

2.2 Equations of motion in the time domain

Substituting the derived expressions for the hydrodynamic reaction forces (2), the equations of motion for the dredge become:

$$\sum_{j=1}^{6} \left\{ \begin{array}{l} (M_{kj} + m_{kj})\ddot{x}_j(t) + \\ \int_{-\infty}^{t} K_{kj}(t-t)\dot{x}_j(t)dt + \\ C_{kj}x_j(t) \end{array} \right\} = F_k(t)$$

$k = 1,...6$

(3)

The contributions to the external forces on the barge, $F_k(t)$, can be divided in:

a. forces due to the mooring system:
   - mooring lines F^{m}
   - spud-pole F^{sp}
   - cutter ladder F^{cl}

b. exciting forces:
   - wave forces F^{w}
   - current forces F^{c}
   - wind forces F^{wd}

c. other forces, among which:
   - viscous damping F^{vs}

Hence:

$$F_k(t) = F_k^{m}(t) + F_k^{sp}(t) + F_k^{cl}(t) + F_k^{w}(t) + F_k^{c}(t) + F_k^{wd}(t) + F_k^{vs}(t)$$

all of these forces may be non-linear functions of time and position of the barge, as will be discussed next.

3. EXTERNAL FORGES

3.1 Mooring lines

Cables undersea, connected to a floating object, meet several external forces, e.g.:

- forces caused by the mass and displacement of the cable,
- hydrodynamic loads due to cable motions, waves and current, and
- friction forces by the bedding of the cable at the seabed.

The dynamic part in the cable force increases with the amplitude and frequency of oscillation at the point of suspension. However at small amplitudes and motions at normal wave frequencies this contribution into the total cable force is small [6]. Hence in case of the cutter suction dredge the dynamic part of the cable force may be neglected.

When neglecting the second order terms, the static cable equations and boundary conditions result in a set of coupled, non-linear first order differential equations (un-elastic cable) [7]; see Figure 2.

$$\frac{dT}{ds} = W \cdot \sin f - R_s$$
$$T \cdot \cos f \cdot \frac{df}{ds} = -Rf$$
$$T \cdot \frac{df}{ds} = W \cdot \cos f - R_f$$
$$\frac{dx}{ds} = \cos f \cdot \cos q$$
$$\frac{dy}{ds} = \cos f \cdot \sin q$$
$$\frac{dz}{ds} = \sin f$$
(4)

The non-linear cable elasticity can be taken into account by adding the elongation of the cable to the length of the unloaded cable. The elastic behaviour of the cable is approximated by a parabolic relation between tension and unit elongation.

For the hydromechanic forces $R_{s,f,q}$ the formulation as given by Wilson [8] is used, which results in:

$$R_s = 0.5 \cdot r \cdot D \cdot c_s \cdot p \cdot v_s \cdot |v_s|$$
$$R_f = 0.5 \cdot r \cdot D \cdot c_N \cdot v_f \cdot \sqrt{v_f^2 + v_q^2}$$
$$R_q = 0.5 \cdot r \cdot D \cdot c_N \cdot v_q \cdot \sqrt{v_f^2 + v_q^2}$$

in which:

$$D = D_0 \cdot (1 - ne)$$

When the lower end of the cable lies on the sea bottom, an adaption is made for the apparent point of anchoring and apparent cable length.

The set of implicit Equations (4) can be solved for the boundary conditions specified at the two ends of the cable. For a given set of initial conditions these equations are numerically integrated using a fourth order Runge-Kutta method. The solution is obtained iteratively, using a "shooting method".

The unknown initial conditions at $S = S_A$ are estimated and from these the boundary conditions at $S = S_B$ are calculated and compared with the actual conditions. A Newton-Raphson iteration procedure is used to generate new initial conditions, until the boundary conditions at $S = S_B$ are satisfied [9].

It was found that for relative high pre-tensioning of the cables, the hydromechanic forces are of minor importance under normal external conditions. If this is the case the direction and magnitude of the cable force is calculated with two-dimensional cable equations, neglecting hydromechanic forces. This implies that $R_{s,f,q} = 0$ and $q = 0$ in Equation (4).

In the DREDMO program the direction and magnitude of the cable force is calculated in advance for a number of possible positions of the point of suspension of each cable around its position at $t = 0$. In the time domain computation the calculation of the instantaneous values for each cable is performed by a simple Lagrange interpolation procedure between these points.

3.2 The cutter ladder

The ladder imposes boundary conditions on the motions of the barge by means of the hinge coupling between both bodies and the hoisting wires.. Due to its large inertia and freedom of motion relative to the barge, the cutter ladder is incorporated as a separate body of the system.

Using Newton's law, the equations of motion become:

$$\sum_{j=1}^{6} \left\{ M_{kj}^{\ell} \cdot \ddot{x}_j^{\ell}(t) \right\} = F_k^{\ell}(t)$$
(5)

in which the superscript ℓ indicates that the values are related to the ladder.

The external force $F_k^{\ell}(t)$ contains several contributions, i.e.:

- soil reaction forces on the cutter, $F^{\ell s}$
- wave and hydrodynamic reaction forces, $F^{\ell w}$
- current forces, $F^{\ell c}$
- hoisting wires forces, $F^{\ell h}$
- forces in the coupling with the barge, $F^{\ell A}$
- underwater weight of the ladder, $F^{\ell z}$.

Forces acting on a rotating cutter which is subjected to an oscillating motion, such as imposed by the motions of the barge in a seaway, will depend on a large amount of parameters, such as:

- soil characteristics
- imposed motion, e.g. surge, sway or heave
- cutter characteristics
- direction of sway
- etc.

The resulting relations between motions of the cutter and the soil reaction forces will be strongly non-linear. In the DREDMO program any desired relation between motions and resulting forces can be specified. Little is known however about the actual relations as function of the relevant parameters. The first results out of the extensive research program at the Delft University of Technology on the soil reaction characteristics on an oscillating cutter are given by De Koning et al. [2].

For most cutter ladders the schematization of the body to a closed, cylindrical construction will be acceptable. If the diameter / wavelength ratio does not exceed a value of about 0.15 and the wave height / diameter ratio is less than 1.0, the diffraction forces become negligible and the well-known Morison formulation for the hydrodynamic loads can be used.

$$F^{\ell w} = F_M + F_D$$
$$= \int_0^{\ell} \left\{ C_M \, r \, \frac{\nabla}{d\ell} \cdot \dot{v} + C_D \frac{1}{2} \, r \, \frac{A}{d} \cdot v \cdot |v| \right\} d\ell \qquad (6)$$

In the mentioned parameter region the inertia forces are predominant $(F_M >> F_D)$. The contribution of the acceleration of the ladder to the relative acceleration between ladder and water will be small compared with the orbital accelerations and hence this hydrodynamic reaction force will be neglected.

The local wave elevation at the ladder location is obtained from the wave elevation at the center of gravity of the barge taking into account its speed of propagation.

Determination of the orbital acceleration in an irregular seaway is treated in the same way as described for the wave forces on the barge. See Chapter 3.4.

If the ladder geometry is such that the wave diffraction forces on the ladder are pre-dominant, these forces are calculated in the frequency domain using a three-dimensional linear potential diffraction theory. Forces in an irregular sea are generated as described for the barge. Hydrodynamic reaction forces, if applicable, are in this case kept constant. The in this way created time series of wave forces serve as input for the time domain calculation.

The Equations of motion (5) now become:

$$\sum_{j=1}^{6} \left\{ M_{kj}^{\ell} \cdot \ddot{x}_j^{\ell}(t) \right\} = F_k^{\ell A}(t) + F_k^{\ell s}(t) + F_k^{\ell h}(t) + F_k^{\ell w}(t) + F_k^{\ell c}(t) + F_k^{\ell z}(t) \qquad k = 1,...6 \qquad (7)$$

As a result of the hinge coupling between ladder and barge this set of equations can be reduced to one degree of freedom.

This results in the implicit equation:

$$\ddot{x}_5^{\ell} = f\left(x_j, \dot{x}_j, \ddot{x}_j, x_5^{\ell}, \dot{x}_5^{\ell}, F^{\ell w}, F^{\ell c}\right)$$

In order to solve this second order differential equation with a finite difference method, it is reduced to a set of first order differential equations. To solve this set of stiff differential equations, use is made of the "theta-method" [11]. This leads to a set of non-linear equations in the unknown variables $x_5^\ell(t)$ and $u(t) = \dot{x}_5^\ell(t)$. This is solved with the Newton-Raphson iteration method. Because of its reduced size (2x2), the required inversion of the Jacobian matrix can be done analytically. The necessary initial prediction vector for the iteration process is delivered by a second order Adams Basforth method [11].

Having solved the equations of motion for the cutter ladder, the reaction forces on the barge can be calculated.

3.3 The spud pole

The loads acting on a spud pole are caused by (see Fig. 4):

- forces and moments in the spud keeper,
- soil reaction forces,
- hydrodynamic forces, and
- mass and buoyancy forces.

The dynamic behaviour of the spud pole is neglected because of its small mass-stiffness ratio. Friction forces between spud pole and spud keepers are taken into account.

For the reaction forces in the spud keepers the freedom of motion of that part of the spud pole, which has penetrated into the seabed is very important. However little is known on the soil reaction forces on oscillating poles, which have a small ratio between penetration depth and pole diameter.

In case of a penetration depth of less than about 3 meters a pinned situation is supposed. For larger penetration depths the pole is supposed to be partially clamped. For this condition use is made of computational methods, which are originally intended for the calculation of the soil reaction forces on mooring dolphins.

The hydrodynamic loading is determined, using Morison's formula, as given in Equation (6). For typical spud-pole dimensions the contribution of the drag force predominates $(F_D > F_M)$. If applicable the current velocity is added vectorially to the orbital velocity. This calculation method tends to somewhat overestimate the total fluid loading. The resulting forces and moments in the spud keeper are calculated and transferred to the center of gravity of the barge [9, 10].

3.4 Wave exciting forces

Time series of the wave exciting forces on the barge are required as input for the DREDMO program.

The first order components of the wave forces can be obtained from frequency domain calculations. Several computational methods are readily available, ranging from strip theory calculations using sectional values derived with two-dimensional potential theories, to three-dimensional linear potential diffraction programs. For irregular seas the wave forces can be determined using the thus calculated frequency dependent transfer functions between the wave force and wave amplitude, F_k^{wa} / z_a, according to:

$$F_k^w(t) = \sum_{i=1}^{N} \left\{ \left| \frac{F_k^{wa}}{z_a}(w_i) \right|_{ki} z_{ai} \cos\left(w_i t + e_{F_k z_i}\right) \right\}$$

The wave amplitudes z_{ai} are derived from the energy density spectrum describing the desired irregular sea-state, by assuming this to consist of a number of (N) regular wave-components:

$$z(t) = \sum_{i=1}^{N} \left\{ z_{ai} \cdot \cos\left(w_i t + e_{z_i}\right) \right\}$$

$$z_{ai} = \sqrt{2 \cdot S_{zz}(w_i) \cdot \Delta w_i}$$

The phase angles e_{z_i} are chosen randomly, while the frequency interval Δw_i depends on the frequency itself.

In general the low frequency second order drift forces may lead to considerable motion amplification for moored floating objects, if any undamped natural frequency of the moored system lies within this frequency-range. In the case of a cutter suction dredge because of the high stiffness of the mooring system (including spud pole and ladder) this usually tends to be not the case. In principle however, the second order drift force contributions can be incorporated in the wave force time series, used as input for the DREDMO program.

3.5 Current forces

As dredges are frequently operating in tidal regions or river estuaries, current loads on both barge and ladder are important.
At present however, there are no practical computational methods available. Therefore in the computer program use is made of formulas, which incorporate empirical coefficients. For the barge only the current forces in surge, sway and yaw direction are taken into account, i.e.:

$$F_{1,2}^{c} = 0.5 \cdot r v_c^2 T \sqrt{L^2 + B^2} \cdot c_{1,2}(a_c)$$
$$F_{6}^{c} = 0.6 \cdot r v_c^2 T (L^2 + B^2) \cdot c_6(a_c)$$

Here the current velocity v_c is corrected for the motions of the barge. The empirical coefficients $c_{1,2,6}(a_c)$ depend on the angle of attack of the current a_c.
Besides these current loads, the current velocity also influences the wave forces and hydrodynamic reaction forces. However for low current velocities these effects may be neglected.

3.5 Viscous roll damping

The potential part of the total roll damping is included in the retardation function. The viscous part has a non-linear behaviour and is calculated separately and is added to the forces in the right hand side of Equation (2).

4. COMPUTATIONAL SCHEME

After assembling all contributions to the external force vector $F_k(t)$, the equations of motion for the barge, Equation (3), have to be solved. Because of the so-called "stiffness" of these equations, mainly caused by the ladder- and spud pole-reaction forces, much attention has been paid to the numerical solution procedure of this set of equations.
The second order differential equations are reduced to a set of first order non-linear differential equations, which is solved by using a finite difference scheme. Because of its unconditional stability use is made of the "theta-method".
To avoid high computational costs this set of equations is solved using a modified Newton-Raphson iteration method [11]. This implies that the total number of equations can be reduced and the necessary Jacobian matrix need not be determined for each iteration step and not even for each time step, but can be kept constant during the computations until convergence is no longer obtained.

5. RESULTS

5.1 Hydrodynamic reaction forces and wave forces

The added mass and damping coefficients, required for the computation of the retardation functions can be calculated with readily available computer programs. Computational results of several of these programs, applied to the geometry of a cutter suction barge, were verified with model experiments. Oscillation tests and waves forces measurements were performed at the Delft Hydraulics Laboratory for various water depths and compared with computational results [1]. A fair agreement was found between theory and experiments.

5.2 Current forces

The Ship Hydromechanics Laboratory of the Delft University of Technology carried out model experiments on a cutter suction barge including the ladder, in order to get reliable information on the current forces and moments.
The general outline of barge and ladder are given in Figure 5. The full-scale current speed range was 0 to 3 knots.
From these experiments, the values for the empirical coefficients $c_{1,2,6}(a_c)$ were obtained. The values are given in Figure 5 as a function of the angle of attack a_c.
For input in the DREDMO program, these coefficients are transformed into polynomial functions, whose coefficients are determined by a least squares method.

5.3 Computational results

The program is applied to a conventional cutter suction dredge operating in an exposed area. The influence of the soil characteristics on the motion behaviour and forces in the construction was investigated. Three different soil characteristics were used. To illustrate the influence of non-linear soil reaction forces, one computation was executed using linear soil characteristics.

5.3.1 Input

The main dimensions of the dredge are given in Table 1 together with the hydrostatic spring rates. The coupling between ladder and barge is assumed to be conventional.
Because of the severe external conditions, the barge is kept on location by the bowlines and a 'christmas tree' configuration at the stern of the vessel. The total number of mooring lines is 5. The ladder is swayed by means of two swing wires. The configuration is outlined in Figure 8. The position of the attachment of the mooring lines on the vessel and the anchor locations are given in Table 3. The mooring line characteristics are also given in Table 3.
All computations are performed for the same external conditions. The wave condition is defined by a Pierson-Moskowitch spectrum with a significant wave height of 1.0 m and a peak-period of 7.0 s. The wave spectrum is presented in Figure 7. The angle of incidence of the waves is 30^0 from the bow (quartering waves). No current is assumed.
From the hydrodynamic coefficients $a(w)$, $b(w)$, the retardation functions $K_{kj}(t)$ and mass m_{kj} are calculated. The values of m_{kj} are summarized in Table 2. In Figure 6 a number of retardation functions are given, i.e. the diagonal of the matrix K_k.
Because different types of soils are to be simulated, no data can yet be used at this point from Reference 3. Therefore here only an approximated dynamic soil behaviour is used to demonstrate its influence on the motions of the dredge.
In situations where the cutter is actually cutting in all directions, use can be made of the assumption that the specific cutting energy, A_{sp}, is constant, so:

$$A_{sp} = \frac{M_c \cdot w_c}{P}$$

with:

M_c shaft torque,
w_c angular velocity of the cutter, and
P cutting production.

The soil reaction force is now calculated using the assumption that:

$$\frac{R \cdot F_k^{\ell s}}{M_c} = c_k$$

where:

R cutter radius

c_k constant, function of the wear of the cutter teeth and shape of the cut profile.

The soil reaction forces are approximated by:

$$F_k^{\ell s}(t) = c_k \cdot f\left\{(A_{sp}V_h(t)),(s(t)d(t)),R,\left(\frac{V_z(t)}{V_h(t)}\right),\left(\frac{V_p(t)}{V_h(t)}\right)\right\} \quad (8)$$

with:

V_h swing velocity

s penetration depth in axial direction

d penetration depth in radial direction

V_p penetration velocity, axial direction

V_z penetration velocity, radial direction.

All these variables are a function of swing direction of the cutter, type of cutter and type of soil. The used (mean) values of the important variables are summarized in Table 4 for different type of soils, i.e. packed sand and soft rock.

5.3.2 Results

The DREDMO program produces time series of the motions of the barge and cutter ladder and of the forces in mooring lines, spud pole keeper, cutter head, side swing wires, hoisting wires and the coupling between ladder and barge. All these time series are plotted and/or statistically and spectrally analysed.

The results of the computer runs given here are primarily intended to demonstrate the capabilities of the program. Because the used soil characteristics are chosen rather arbitrarily, no definite conclusions should be drawn from the behaviour of the dredge in the two types of soil. However some interesting features can be observed.

The motions of the center of gravity of the barge are given in Table 5.

The results show an increase in surge motion for test 3, mainly caused by larger motion amplitudes in the negative x_1-direction while no significant differences occur for sway and heave. However because of the chosen formulation of $F_k^{\ell s}(t)$ in Equation (8), the used combination of increasing A_{sp}-value with reduced swing velocity in case of soft rock. Although correct as such, also diminishes the relative differences in behaviour of soils between sand and rock in the x_1, x_3 plane.

Samples of time series of the motions of the barge are given in Figure 9. Especially for surge motion the subharmonic behaviour is seen to be significant. This is caused by the non-linear characteristics of the restoring forces, i.e. the soil reaction forces. That this is actually the case is clearly demonstrated when comparing the results with those of the test with linear soil characteristics (Test 2); see Figure 10. No significant sub-harmonic response occurs. These observed phenomena of sub-harmonic motions are more generally known in connection with moored ships [12]. In Table 6 the forces on the cutter are given. As mentioned before, the used formulation for the soil behaviour tends to underestimate the differences. The maximum penetration velocity in axial direction is also given. Here a pronounced difference can be observed.

In Figure 11 sample time series for the soil reaction forces $F_k^{\ell s}(t)$ in Test 3 are given. This illustrates the non-linear responses. It can be seen that for this condition the cutter temporarily looses contact with the soil.

This could also give an explanation for the enlarged surge amplitudes in Test 3, because the cutter temporarily looses contact with the soil. The non-linear characteristics of this restoring force change, which is not the case in Test 1. This results in different motion behaviour.

6. CONCLUSIONS

The DREDMO program simulates the dynamic behaviour of a cutter suction dredge in waves. In order to be able to incorporate non-linear forces acting on the system the equations of motion are formulated and solved in time domain. In particular the non-linear soil characteristics have a pronounced influence on the behaviour of the dredge, as demonstrated by the computational results. From these results it is also apparent that this dynamic behaviour depends on the type of sail. Knowledge of the soil reaction forces on an oscillating cutter is thus important. This, however, is still very much a research subject. The DREDMO program is a useful tool both for design studies and for down time assessments. However, the accuracy obtained for downtime calculations very much depends on the availability of data on the particular soil characteristics.

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length	71.80	m
beam	17.75	m
depth	5.40	m
draught (1/2 L)	4.00	m
displacement weight	4290.00	ton
radii of gyration: k_{xx}	6.8	m
k_{yy}	22.2	m
k_{zz}	21.9	m
Metacentre height	4.4	m
position centre of gravity above base	5.0	m
position centre of gravity from APP	37.4	m
hydrostatic spring rates C_{33}	11025	kN/rad
C_{44}	139171	kNm/rad
C_{55}	4257740	kNm/rad
C_{35}	36331	kN/rad
length cutter ladder	41.5	m
position hinge relative to CG barge	22.5	m
mass ladder	513	ton

Table 1: Main Dimensions Cutter Suction Dredge

m_{kj}	1	2	3	4	5	6
1	291	0	0	0	- 3604	0
2	0	860	0	- 3476	0	55
3	0	0	8688	0	- 40165	0
4	0	-3476	0	263356	0	- 33475
5	-3604	0	-40165	0	5116363	0
6	0	55	0	- 33475	0	2555953

Table 2: Added Mass m_{kj}

Line number	Point of attachment			Anchor position			ℓ	EA
	x	y	z	x	y	z		
1	-33.6	0	- 7.3	-233.6	0	-27.1	200.2	85021
2	-33.6	0	- 7.3	- 33.6	-150	-27.1	150.7	85021
3	-33.6	0	- 7.3	- 33.6	+150	-27.1	150.7	85021
6	51.6	0	10.0	162.7	- 77.8	-27.1	140.1	102203
7	51.6	0	10.0	162.7	+ 77.8	-27.1	140.1	102203

Table 3: Mooring Line Characteristics

	packed sand test 1	linear (sand) test 2	soft rock test 3
Asp kJ/m^3	600	600	3000
v_h m/s	0.3	0.3	0.1
c_1	1	-	1
c_3	2	-	2
R m	0.87	0.87	0.75

Table 4: Soil Characteristics for thye Tests 1, 2 and 3

	Significant			Maximum		
	x1	x2	x3	x1	x2	x3
Test 1	0.25	0.40	0.38	0.34	0.54	0.50
Test 3	0.31	0.43	0.37	0.43	0.59	0.48

Table 5: Motion Amplitudes of Center of Gravity of a Barge (m)

	max. crest-trough values soil reaction forces			max. penetration velocity
	F_1^{sl} (kN)	F_2^{sl} (kN)	F_3^{sl} (kN)	Vp (m/s)
Test 1	1120	370	1299	0.20
Test 3	1340	400	1502	0.12

Table 6: Soil Reaction at Cutter

Remark: In Test 3 all minimum soil reaction forces are zero, due to cutter loosing contact with soil.

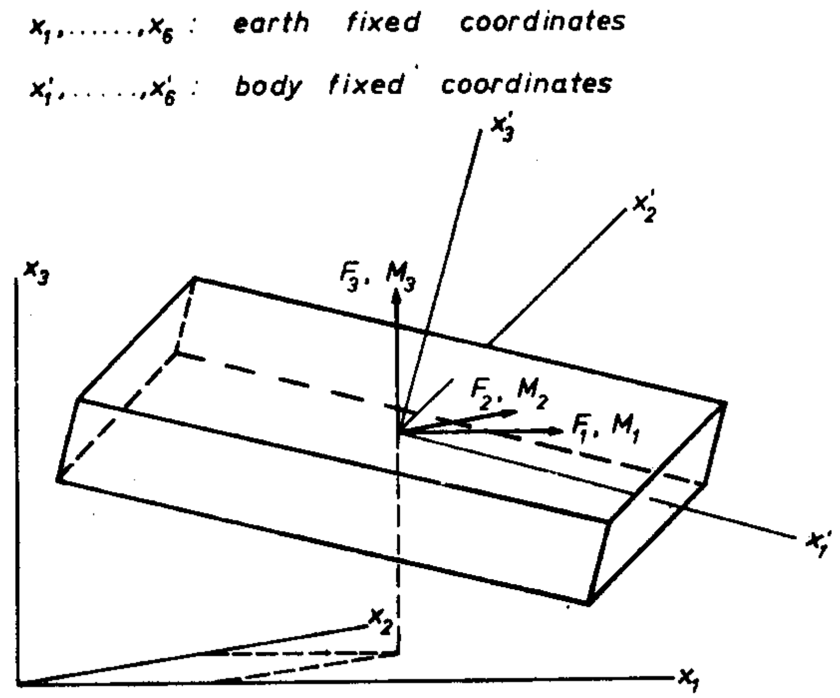

Figure 1: Definition of Coordinate Systems

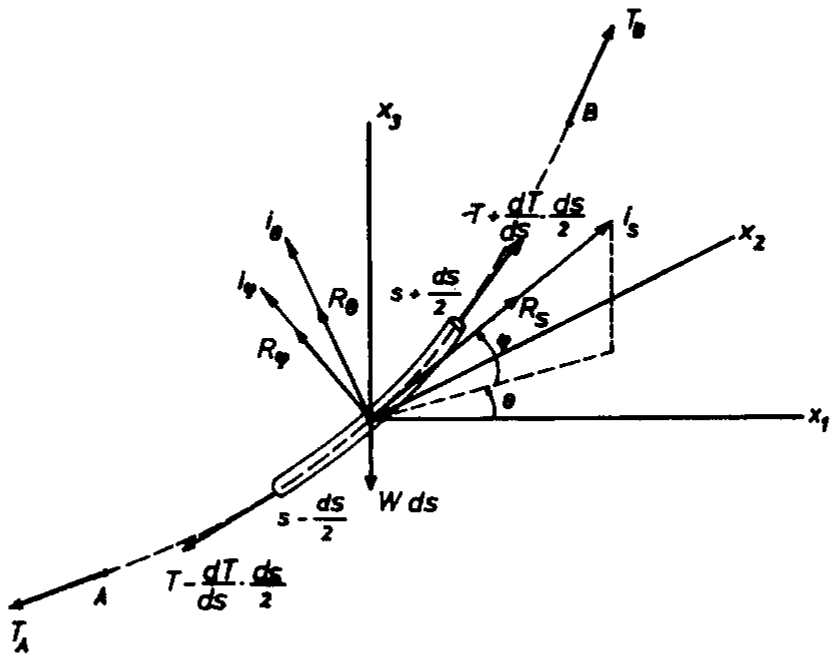

Figure 2: Local Coordinate System for Mooring Line

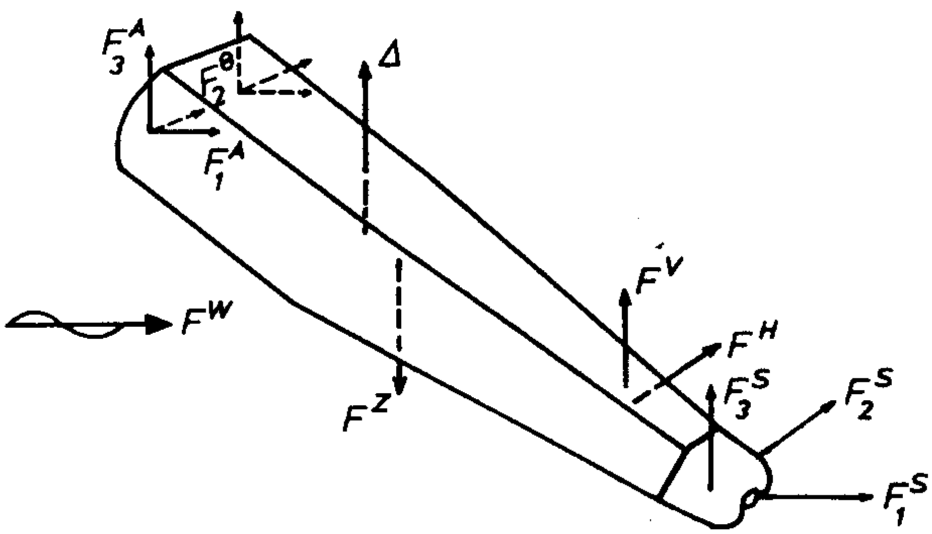

Figure 3: Load Condition Cutter Ladder

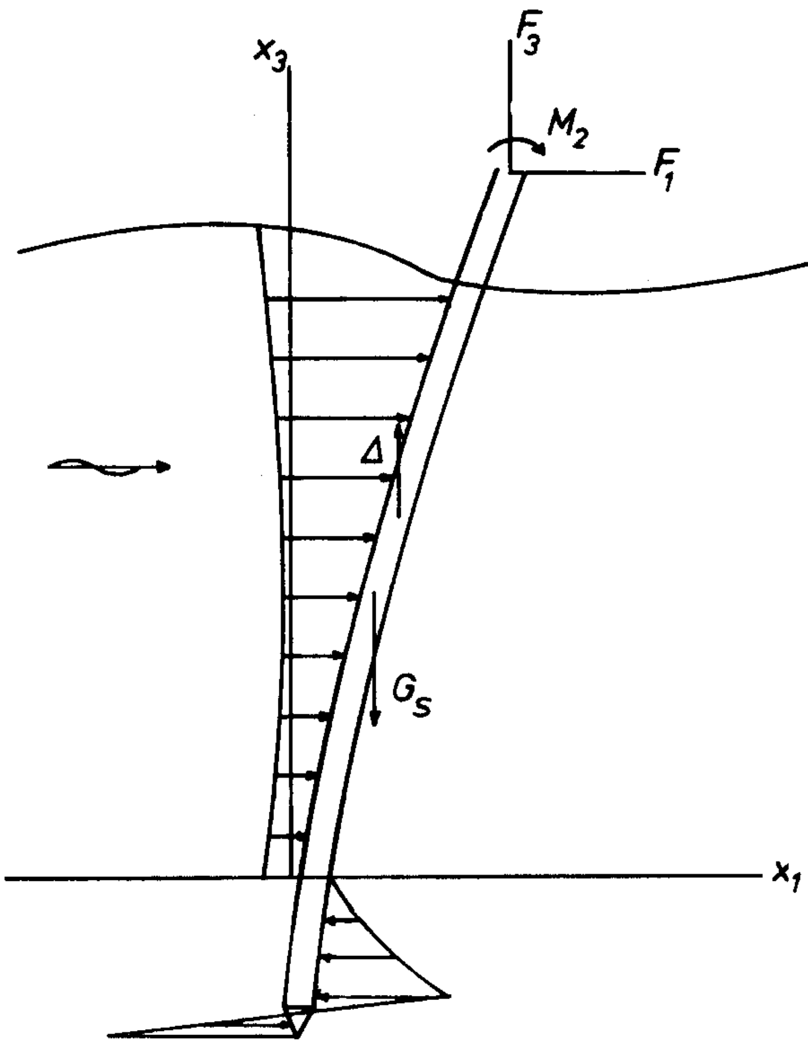

Figure 4: Load Condition Spud-Pole

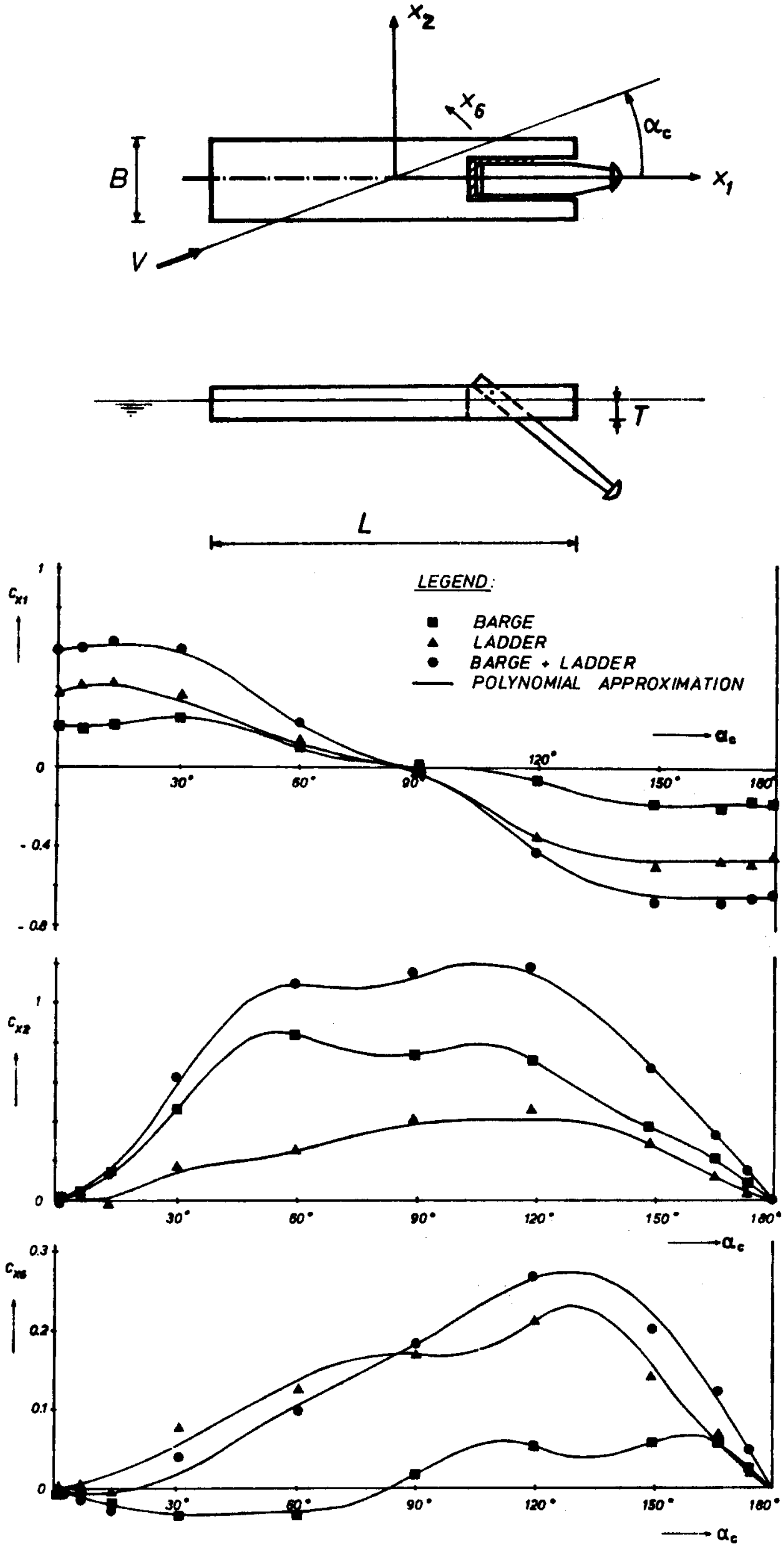

Figure 5: Coefficients of Current Forces and Moment

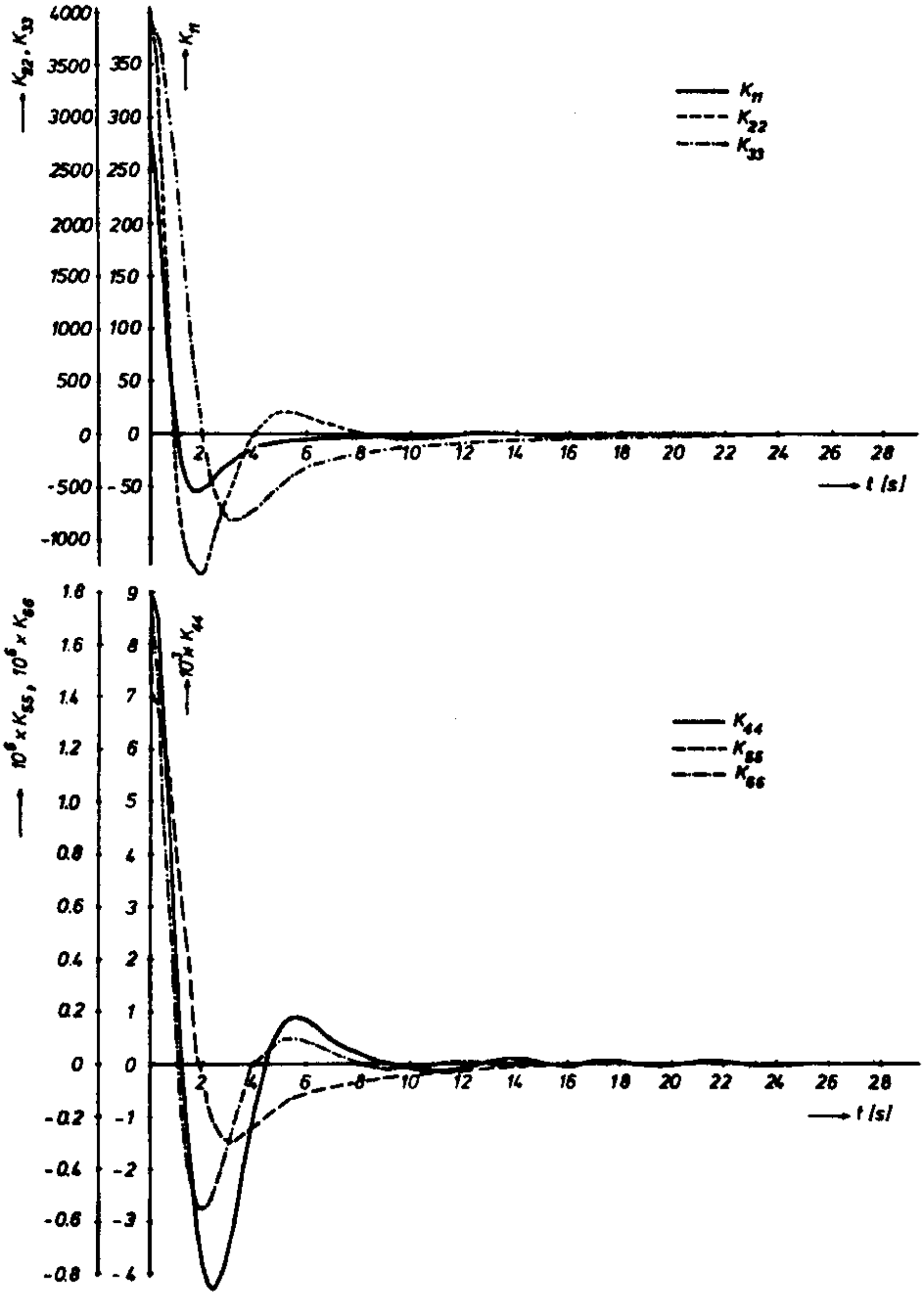

Figure 6: Retardation Functions Cutter Suction Dredge

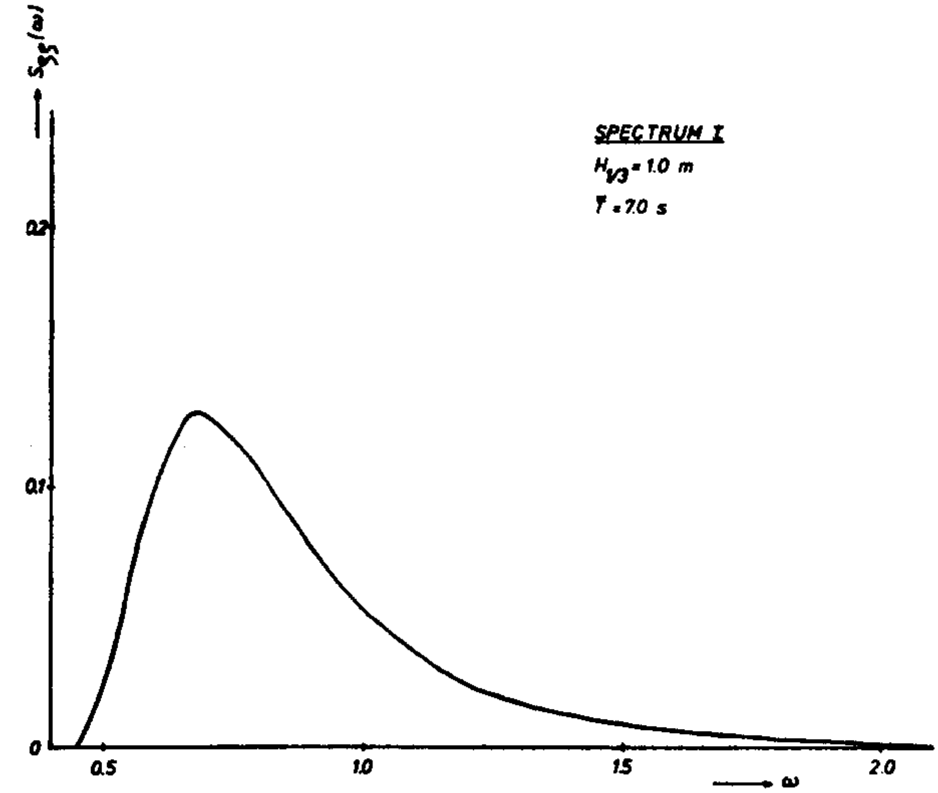

Figure 7: Applied Wave Spectrum

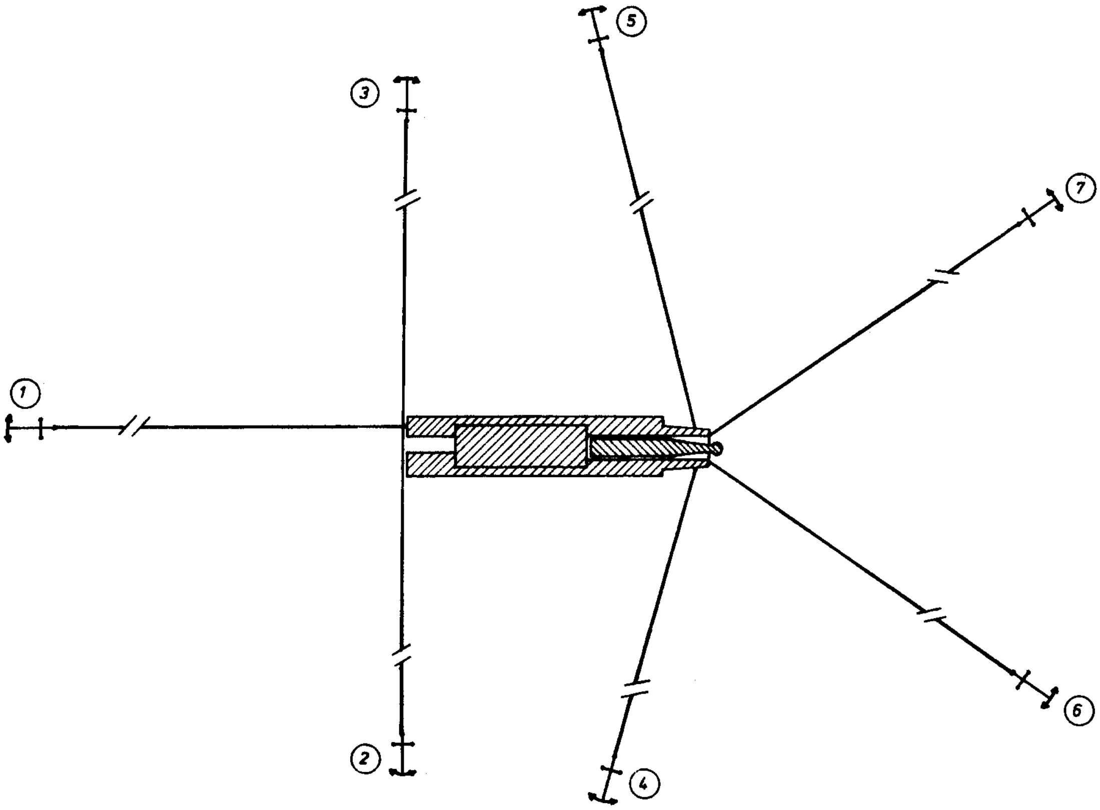

Figure 8: Mooring Line Arrangement for Christmas Tree Configuration

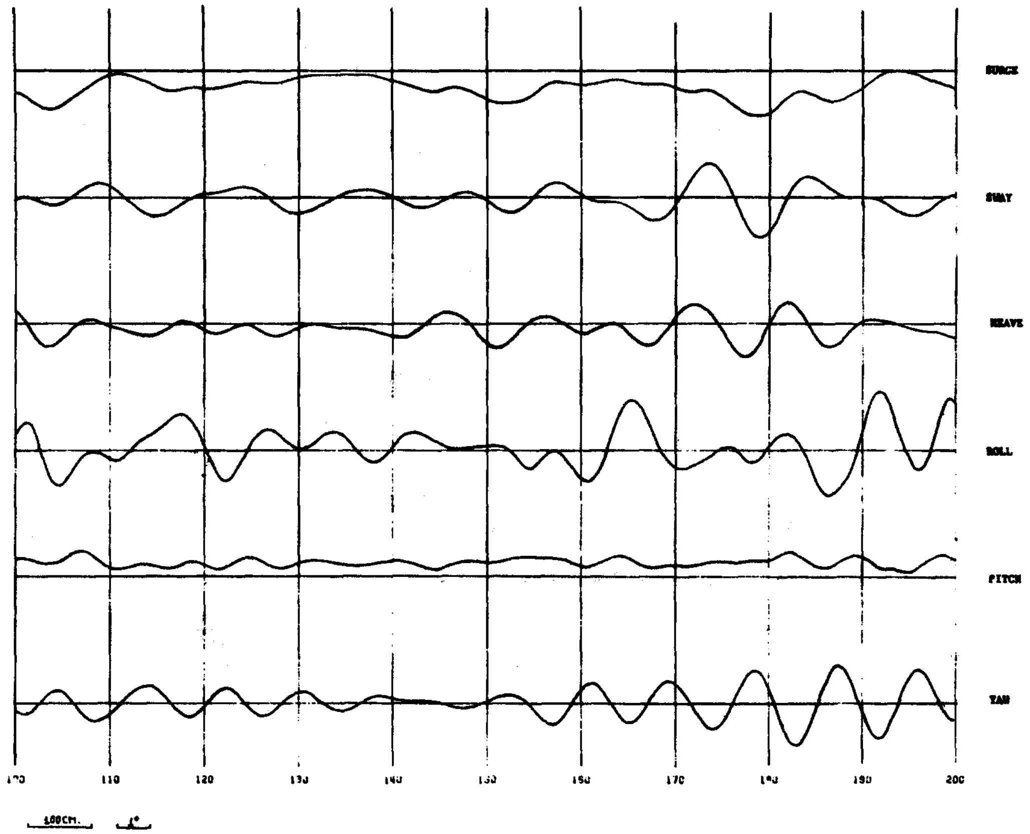

Figure 9: Time Series Motions of the Center of Gravity of a Barge, Test 3

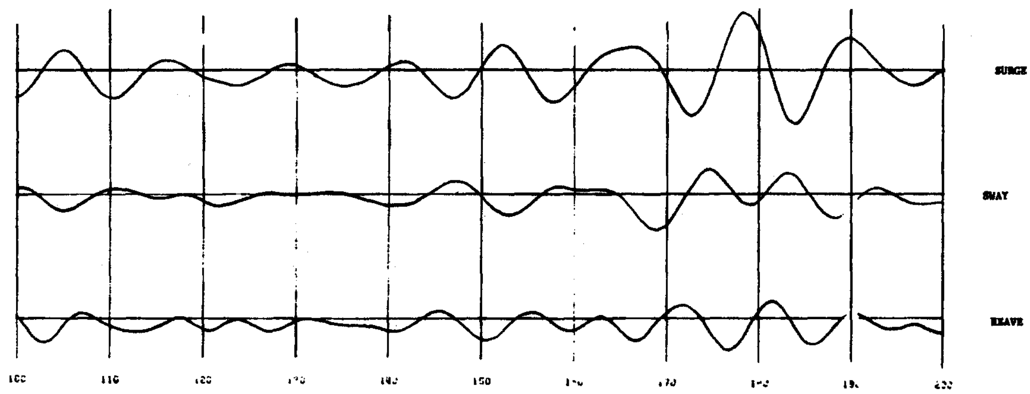

Figure 10: Time Series Motions of the Center of Gravity of a Barge, Test 2

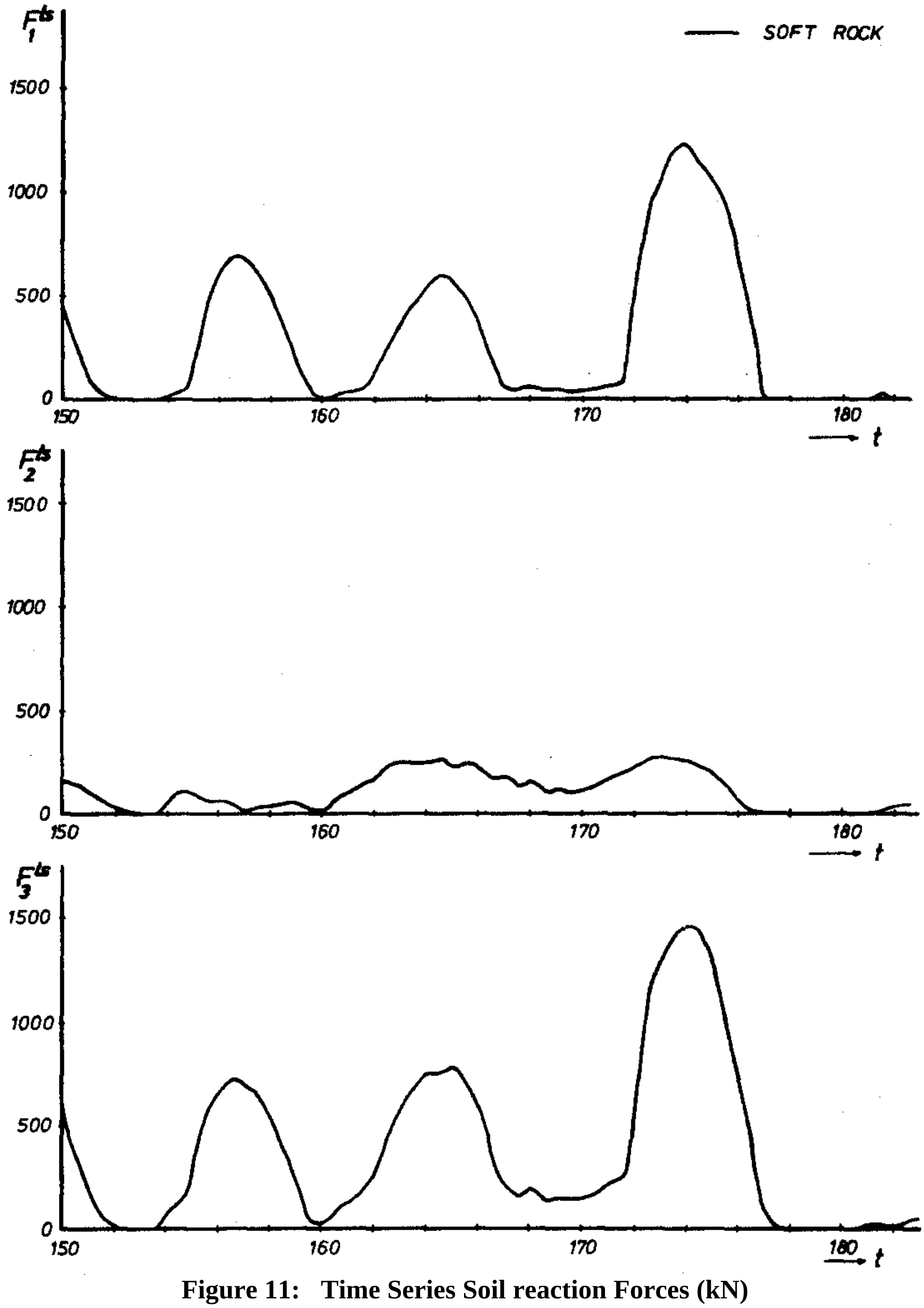

Figure 11: Time Series Soil reaction Forces (kN)