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A Simple Method for Determining the Manoeuvring Indices *K* and *T* from Zigzag Trial Data

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Abstract

Nomoto's first order model is the simplest mathematical model to describe ship manoeuvres. Calculated manoeuvring data have been analysed here to determine the relation between the manoeuvring indices K and T of Nomoto (1960) and zigzag manoeuvring characteristics. The results have been reflected in graphs, which can be used then to determine these indices from actual zigzag manoeuvres. This report is a translation in English of a report in Dutch of the author, Journée (1970), on this topic.

1 Introduction

The horizontal motions of a vessel due to a rudder deflection can be described by using mathematical models. The most simple system, the first order model of Nomoto (1960), is a compromise between the demand for a simple mathematical model and a fair approximation of the actual manoeuvres of the ship.
Nomoto has published some methods to estimate the manoeuvring indices K and T from (full-scale) zigzag trial data. But, using these index estimators, it appears often that the calculated manoeuvres deviate considerably from those measured on the ship. Because of this, a method has been developed to determine these indices in such a way that the differences between calculated and measured data, as far as yaw period and overshoot are concerned, are as low as possible.
Using Nomoto's first order model, a large number of zigzag manoeuvres have been calculated here at a practical range of K and T values. These data have been analysed and the relation between the zigzag manoeuvring characteristics and the Nomoto indices K and T have been reflected in graphs. Reversed, these graphs can be used then to determine these indices from actual zigzag manoeuvres.

2 Equation of Motion

The first order model of Nomoto (1960) reads as follows:

$$T \cdot \ddot{y} + \dot{y} = K \cdot (d - d_r)$$

Equation (1)

In here:

$\ddot{y}$	Yaw angular acceleration
$\dot{y}$	Yaw angular velocity or rate of turn (deg/s^2 or rad/s^2)
y	Yaw angle (deg/s or rad/s)
d	Actual rudder angle (deg or rad)
$d - d_r$	Effective rudder angle (deg or rad)
K	Proportionality constant (1/sec)
T	Time constant (s)

3 Actual and Ideal Kempf Zigzag Manoeuvres

An "ideal" Kempf zigzag manoeuvre has to fulfil the following requirements here:

- equal absolute values of rudder angles,
- equal absolute values of rudder angle velocities and
- "rudder-order" when $y = -d$.

It is obvious that an actual zigzag manoeuvre can't fulfil these specific demands exactly. It is possible however, to transform an actual manoeuvre to an ideal manoeuvre with sufficient accuracy. Therefore, computed ideal Kempf zigzag manoeuvres have been analysed here.

4 Analysis of First Order Model

Using Nomoto's first order model, a large number of zigzag manoeuvres have been calculated at a practical range of K and T values, with the following parameters:

- d_a the level of the rudder angle,
- d_r the rudder angle at which the ship sails a straight course and
- $\dot{d}$ the rate of turn of the rudder.

The following parameters have been obtained from the computed course histories:

- t_p the period,
- y_g the mean course and
- y_a the maximum course deviation relative to y_g.

To avoid transient effects, the third period has been used to determine these magnitudes, see figure 1.

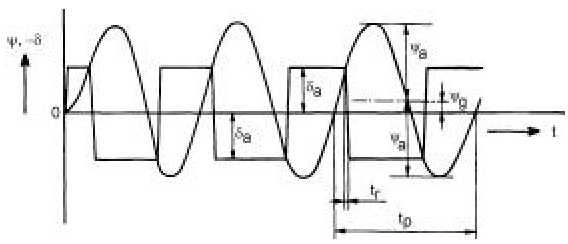

Figure 1 Ideal Kempf Zigzag Test

The following relations have been found by analysing these calculated data:

$$t_p = t_{p_0}\left\{1+\frac{C_2}{\left(\frac{d_a}{d_r}\right)^2-1}\right\}+\frac{1}{2}t_r\cdot C_1$$

Equation (2)

$$\frac{y_a}{d_a} = \frac{y_{a_0}}{d_a} - K\cdot\frac{1}{2}t_r + K\cdot T\cdot d_a^{\,2}\cdot C_3 + \left(\frac{d_r}{d_a}\right)^2\cdot C_4$$

Equation (3)

$$\frac{y_g}{d_r} = C_5 - 1.09\cdot K\cdot\frac{1}{2}t_r$$

Equation (4)

In here:

- y_a, y_g, d_a and d_r in degrees
- t_p, t_{p_0} and t_r in seconds.
- $t_r = 2d_a / |\dot{d}|$ is the time of rudder action
- t_{p_0} is the period for manoeuvres with $|\dot{d}| = \infty$ and $d_r = 0$
- y_{a_0} is the yaw amplitude for manoeuvres with $|\dot{d}| = \infty$ and $d_r = 0$
- t_{p_0}/T, y_{a_0}/d_a, C_1, C_2, C_4 and C_5 are functions of the product $K \cdot T$
- C_3 is a function of $T \cdot |\dot{d}|$

These relations are given in the figures 5, 6 and 7 as will be described further on.

5 Transformation of Actual Zigzag Manoeuvres to Ideal Manoeuvres

The largest deviation of an actual zigzag test from an ideal one is generally caused by not fulfilling the requirement $y_{ex} = -d_0$, see figure 2.

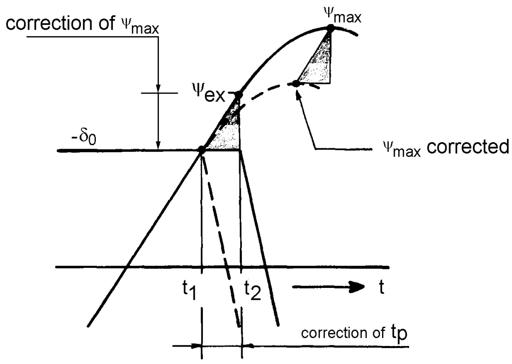

Figure 2 Manoeuvre Transformation

Because of transient effects, the second and the third period of the time history has to be used to determine mean values of d_a, $|\dot{d}|$, y_a, y_g and t_p, being the values for an ideal manoeuvre.

However, when supposing that $\ddot{y} = 0$ at $t = t_1$, two additional corrections have to be used to obtain the values y_a and t_p for an ideal manoeuvre:

$$y_{\max\text{-ideal}} = y_{\max} - (y_{ex} - (-d_0))$$
$$t_{p\text{-ideal}} = t_p - (t_2 - t_1)$$

6 Determination of *K* and *T* from a Zigzag Trial

As the "ideal" values of d_a, $|\dot{d}|$, y_a, y_g and t_p are known, the unknowns in the three equations (2), (3) and (4) are K, T and d_r. These equations can be solved in an iterative way.

Doing this "manually", is very time consuming. Without a large influence on the accuracy however, these equations can be simplified by assuming that:

$$d_r << d_a$$
$$C_1 \approx 4 - 0.25 \cdot K \cdot T$$
$$C_3 \approx 0$$

Then, equations (2) and (3) reduce to:

$$t_{p_0} = t_p - 2 \cdot t_r + 0.125 \cdot K \cdot T \cdot t_r$$

Equation (5)

and

$$\frac{y_{a_0}}{d_a} = \frac{y_a}{d_a} - \frac{1}{2} \cdot K \cdot t_r$$

Equation (6)

From derivations in the Appendix follows:

$$T = \frac{t_{p_0}}{4 \cdot \left(\frac{-1}{K \cdot T} + l \right)} = \frac{t_p - 2 \cdot t_r + 0.125 \cdot t_r \cdot K \cdot T}{4 \cdot \left(\frac{-1}{K \cdot T} + l \right)}$$

Equation (7)

In equation (7) is l a function of $K \cdot T$ only, see figure 4. Also follows from the derivations in the Appendix that y_{a_0} / d_a is a function of $K \cdot T$ only, see figure 4.

Thus, with known "ideal" values of y_a, d_a, t_p and t_r, the coefficients K and T can be found from equations (5), (6) and (7) by a simple and fast iteration.

The following procedure provides the coefficients K and T in a very quick and simple way:

1. As a first guess for y_{a_0} / d_a: ignore $0.5 \cdot K \cdot t_r$ in equation (6).

2. At y_{a_0} / d_a, figure 4 provides now $K \cdot T$ and l.

3. Equation (7) provides T.

4. From $K \cdot T$ and T follows K.

5. Using equation (6), this procedure will be repeated from step 2 with a new guess for y_{a_0} / d_a.

The procedure will be terminated as K and T do not change anymore.

7 Acknowledgement

The author is very grateful to Mr. G. van Leeuwen for his support when solving mathematical problems during the analysis of this first order system.

8 References

Journée (1970)
J.M.J. Journée, *Een eenvoudige methode ter bepaling van de manoeuvreer-indices K en T uit zig-zag proeven*, Report 267, 1970, Ship Hydromechanics Laboratory, Delft University of Technology, The Netherlands.

Nomoto (1960)
K. Nomoto, *Analysis of Kempf's Standard Manoeuvre Test and Proposed Steering Quality Indices*, First Symposium on Ship Manoeuvrability, DTRC Report 1461, October 1960.

9 Appendix

From the equation of motion, given in equation (1), it can be found for $|\dot{d}| = \infty$ and $d_r = 0$ (see figure 3):

$$\dot{y} = \dot{y}_0 \cdot e^{-t/T} + K \cdot d_0 \cdot \left(1 - e^{-t/T}\right)$$

Equation (8)

and

$$y = y_0 + \dot{y}_0 \cdot T \cdot \left(1 - e^{-t/T}\right) - K \cdot T \cdot d_0 \cdot \left(1 - e^{-t/T} - t/T\right)$$

Equation (9)

From equations (8) and (9) follows:

$$y = y_0 + \dot{y} \cdot T \cdot \left(e^{-t/T} - 1\right) - K \cdot T \cdot d_0 \cdot \left(e^{-t/T} - 1 - t/T\right)$$

Equation (10)

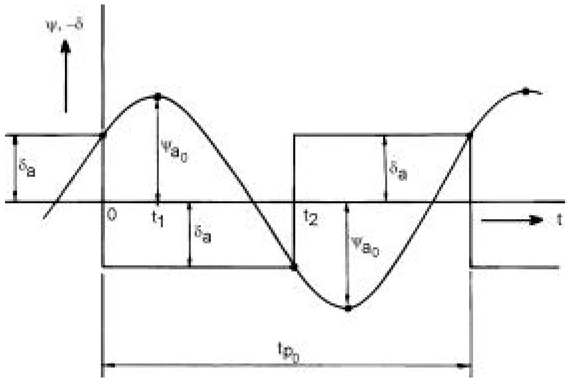

Figure 3 Infinite Rudder Rate of Turn

Values at t_2:

From equation (8) follows:

$$\dot{y}_2 = \dot{y}_0 \cdot e^{-t_2/T} + K \cdot d_a \cdot \left(1 - e^{-t_2/T}\right)$$

Ignoring the transient effects yields:

$$\dot{y}_2 = -\dot{y}_0$$

Thus:

$$\frac{\dot{y}_0}{d_a} = -K \cdot \frac{1 - e^{-t_2/T}}{1 + e^{-t_2/T}}$$

Equation (11)

From equation (9) follows:

$$y_2 = y_0 + \dot{y}_0 \cdot T \cdot \left(1 - e^{-t_2/T}\right) - K \cdot T \cdot d_a \cdot \left(1 - e^{-t_2/T} - t_2/T\right)$$

Because $y_0 = +d_a$ and $y_2 = -d_a$ is:

$$\frac{\dot{y}_0}{d_a} = \frac{-2 + K \cdot T \cdot \left(1 - e^{-t_2/T} - t_2/T\right)}{T \cdot \left(1 - e^{-t_2/T}\right)}$$

Equation (12)

From equations (11) and (12) follows:

$$\frac{t_2}{T} = \frac{-2}{K \cdot T} + 2 \cdot \frac{1 - e^{-t_2/T}}{1 + e^{-t_2/T}}$$

When renaming: $\frac{1 - e^{-t_2/T}}{1 + e^{-t_2/T}} = l$, then l is a function of $\frac{t_{p_0}}{T}$, because: $t_{p_0} = 2 \cdot t_2$.

Then the following relation appears for t_{p_0}:

$$\frac{t_{p_0}}{T} = \frac{-4}{K \cdot T} + 4 \cdot l$$

Equation (13)

From the equations above follows that l, $\frac{t_2}{T}$ and $\frac{t_{p_0}}{T}$ are functions of the product $K \cdot T$.

In figure 5, t_{p_0} has been plotted against T with K as parameter. Equation (13) and this figure show that $t_{p_0} = \frac{-4}{K}$ for $T = 0$ and that $t_{p_0} = \infty$ for $K = 0$.

Values at t_1:

From equation (8) follows:

$$\dot{y}_1 = \dot{y}_0 \cdot e^{-t_1/T} + K \cdot d_a \cdot \left(1 - e^{-t_1/T}\right)$$

Because $\dot{y}_1 = 0$ is:

$$\frac{\dot{y}_0}{d_a} = -K \cdot \left(e^{t_1/T} - 1\right)$$

Equation (14)

Then, from equations (11) and (12) follows that:

$$l = \frac{1 - e^{-t_2/T}}{1 + e^{-t_2/T}} = e^{t_1/T} - 1$$

Because l is a function of the product $K \cdot T$, also t_1/T will be a function of $K \cdot T$.

From equation (10) follows that:

$$y_1 = y_0 - K \cdot T \cdot d_a \cdot \left(e^{t_1/T} - 1 - t_1/T\right)$$

Thus, because $y_0 = +d_a$ and $y_1 = y_{a_0}$ is:

$$\frac{y_{a_0}}{d_a} = 1 - K \cdot T \cdot \left(e^{t_1/T} - 1 - t_1/T\right)$$

Equation (15)

Because t_1/T is a function of $K \cdot T$, also $\frac{y_{a_0}}{d_a}$ will be a function of $K \cdot T$.

Figure 6 shows $\frac{y_{a_0}}{d_a}$ as a function of $K \cdot T$. Equation (15) and this figure show that $\frac{y_{a_0}}{d_a} = 1$ for $T = 0$ and that $\frac{y_{a_0}}{d_a} = 0$ for $K = 0$.

However, from the previous follows too that both $K \cdot T$ and l are functions of $\frac{y_{a_0}}{d_a}$.

In figure 4, $\frac{-1}{K \cdot T}$ and l have been plotted against $\frac{y_{a_0}}{d_a}$.

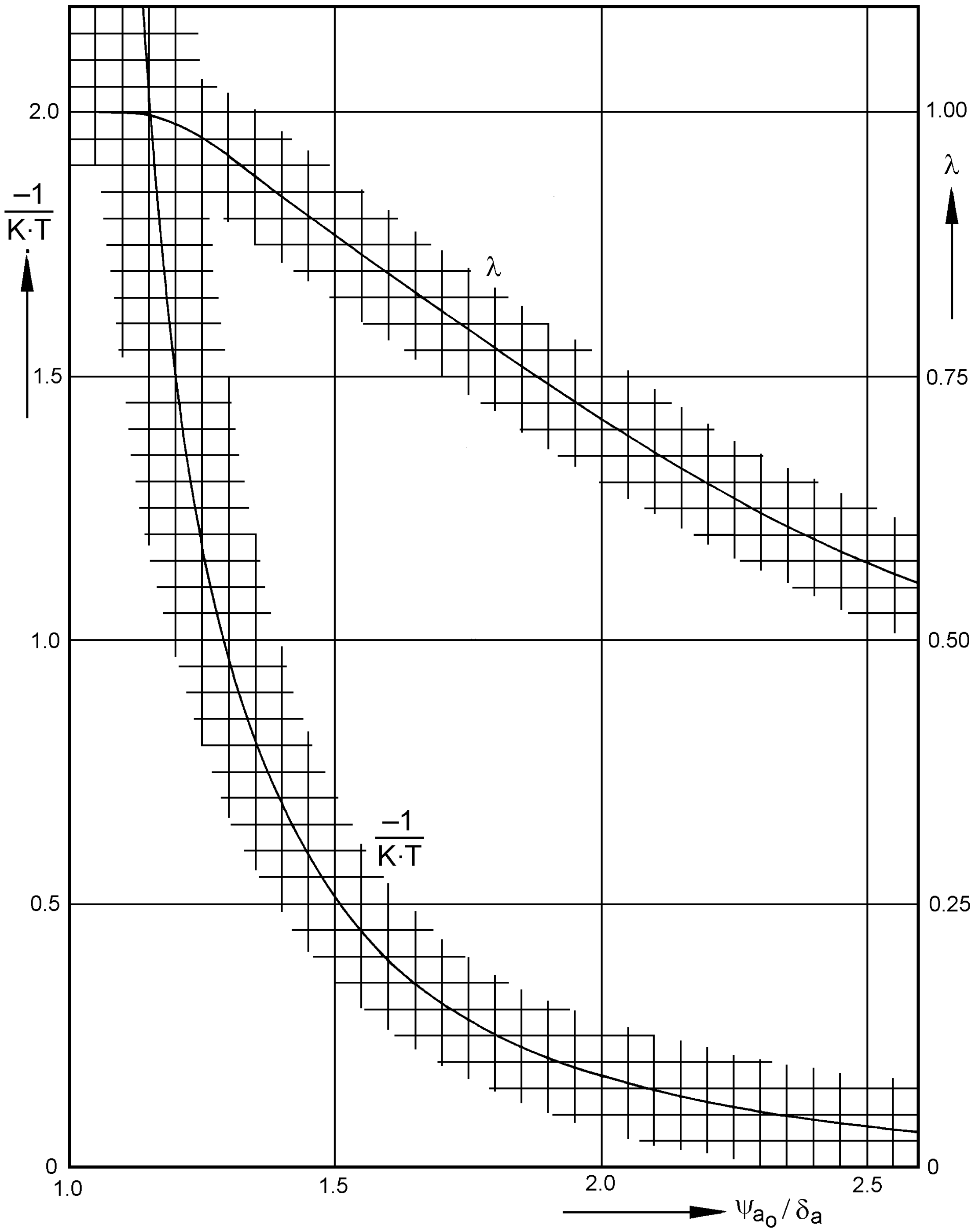

Figure 4 Parameters l and $\frac{-1}{K \cdot T}$ as a Function of $\frac{y_{a_0}}{d_a}$

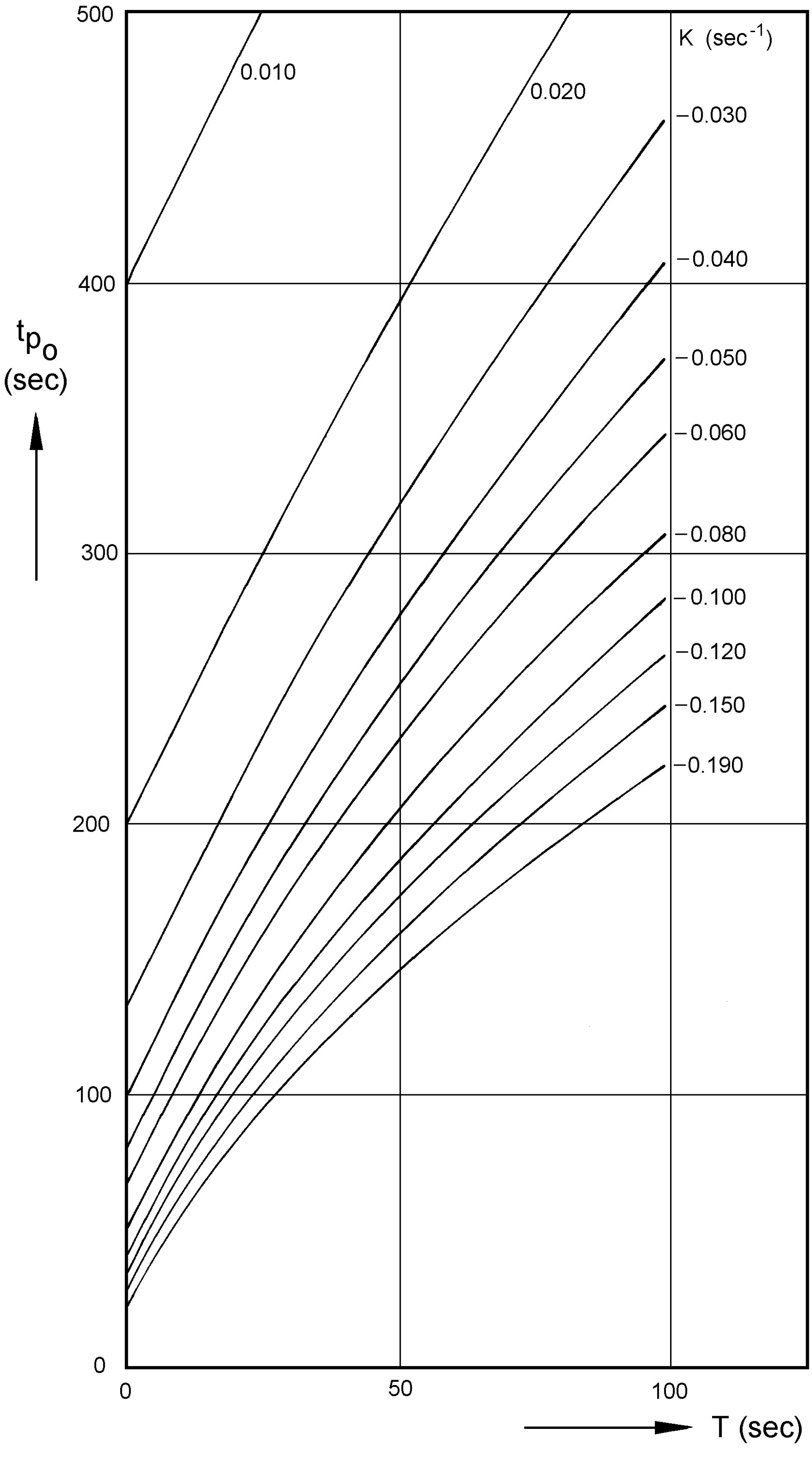

Figure 5 Period t_{p_0} as a Function of K and T

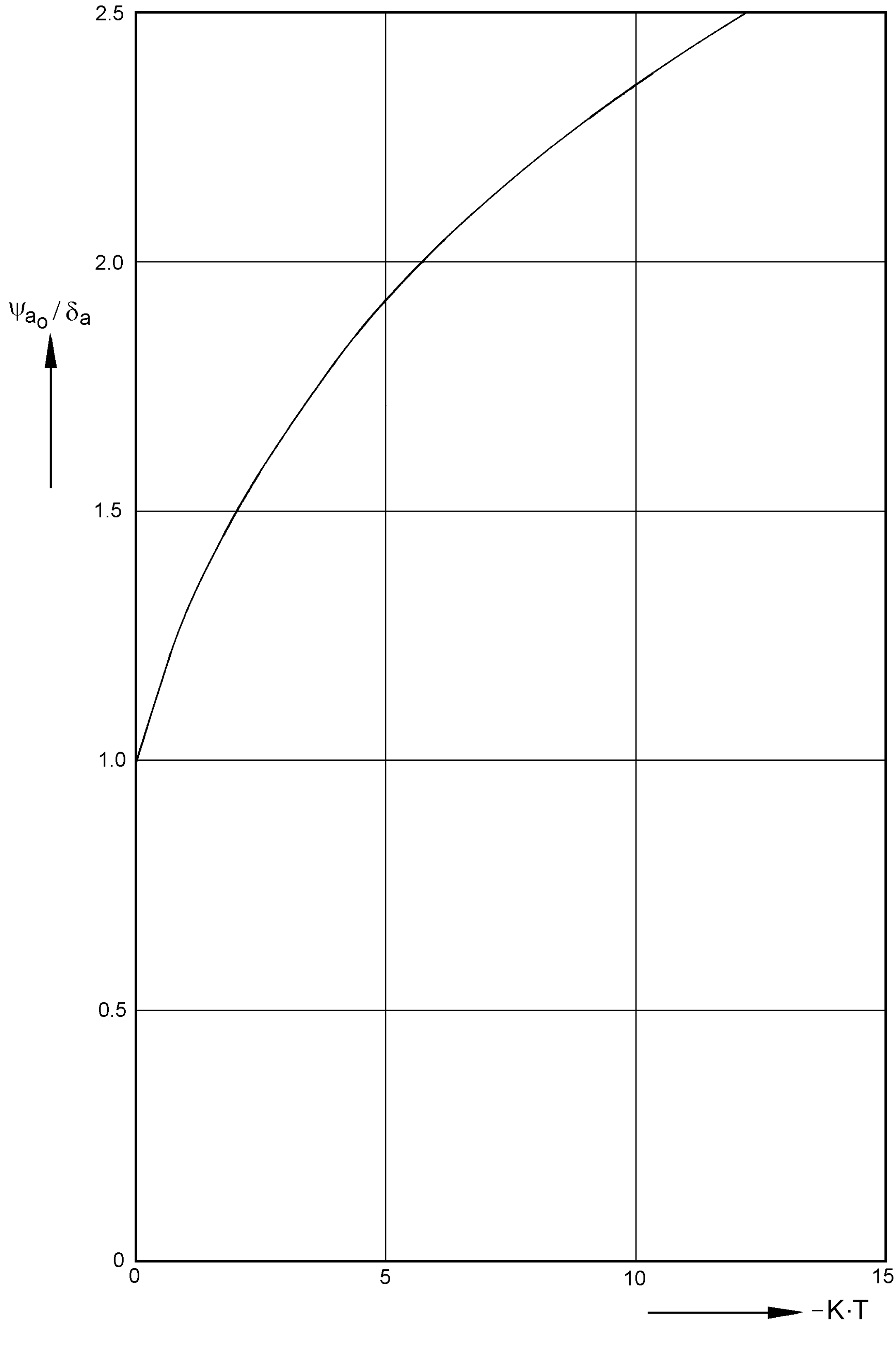

Figure 6 Transfer Function $\frac{y_{a_0}}{d_a}$ as a Function of $K \cdot T$

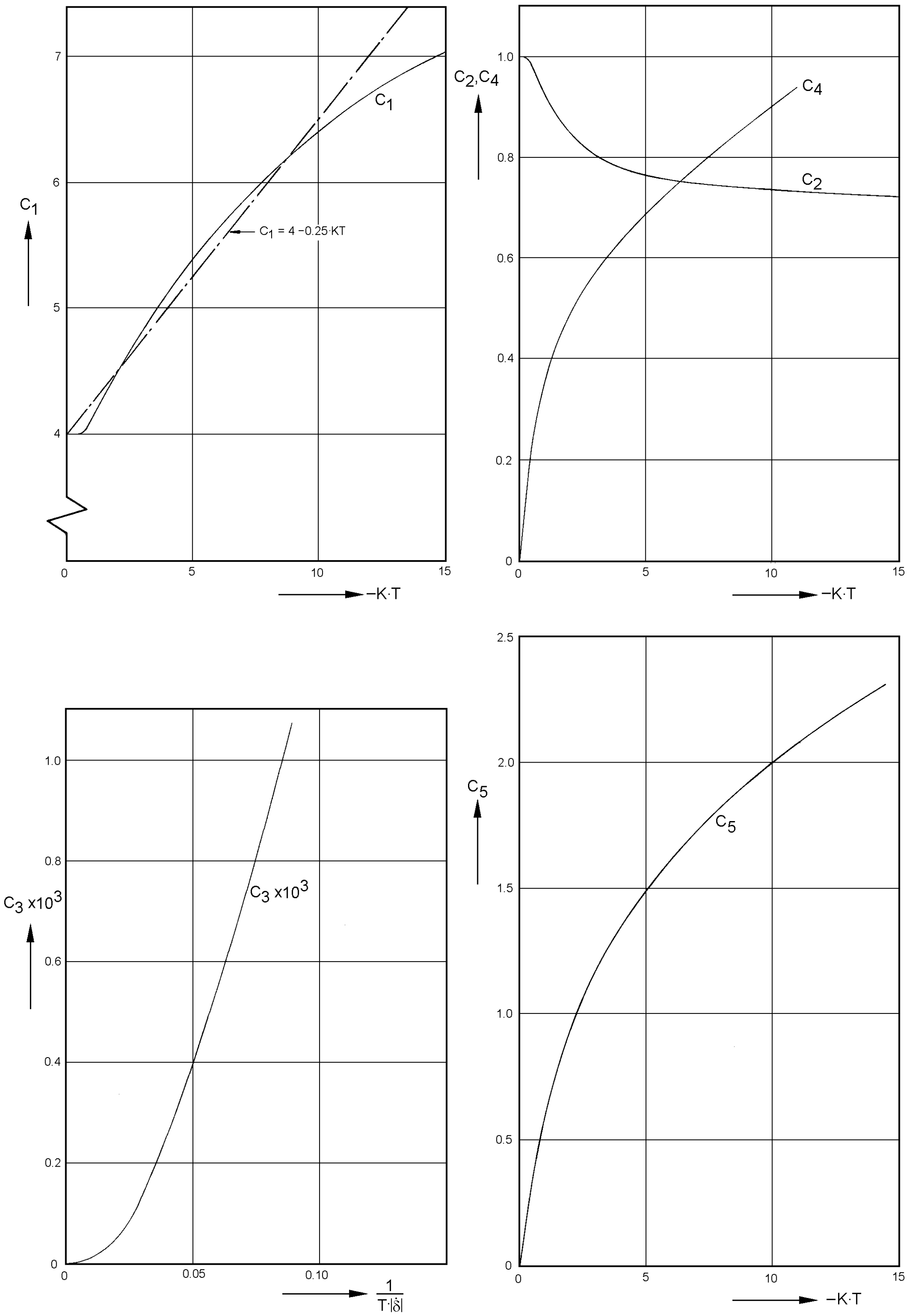

Figure 7   Correction Coefficients